Mathematics

ANALYSIS AND APPROACHES SL

EXAM PRACTICE WORKBOOK

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Introduction

Revising for exams can sometimes be overwhelming. This book is designed to help you structure your revision and provide opportunities to practise sitting exam-style papers. Revision should be a cycle of going through your notes and textbooks, practising exam-style questions, reviewing your strengths and weaknesses, and returning to your notes and textbooks.

There are five skills that are needed for exam success:

- knowledge of all the topics required in the syllabus
- facility with basic algebra and arithmetic
- familiarity with your calculator
- the ability to make links and solve problems
- calmness under test conditions.

You need to be aware of your own strengths and weaknesses in each of these areas. This revision guide is designed to help with each skill.

How to use this book

The book comprises four sections that are designed to help you master the five skills listed above.

Calculator checklist

This lists all the tools provided by your GDC (graphical display calculator) that you need to be familiar with. Different calculators might have slightly different input methods, so it is best to use your own calculator manual (these can be found online) to find out the exact syntax yours uses.

Syllabus revision

This section goes through the syllabus line by line to make sure you have covered every part thoroughly. Each skill required in the syllabus is exemplified by a question. You can either start by going over the syllabus content, or by doing the questions. These questions illustrate the basic skills you need; however, they are not all designed to be exam-style questions as they are designed to check syllabus understanding rather than problem-solving. The answers to these questions can be found online at www.hoddereducation.co.uk/ib-extras. Once you are happy, tick the 'revised' box. If you need more details, there are references to the section in the accompanying Hodder Education *Mathematics for the IB Diploma: analysis and approaches SL* textbook corresponding to each syllabus item.

Questions with calculator icons are designed specifically to test calculator or non-calculator skills. Those without an icon could be done either with or without a calculator.

Paper plan

This table provides an overview of the entire syllabus that maps the practice papers in this book and in the *Mathematics for the IB Diploma: analysis and approaches SL* textbook to the different topics and also serves as a revision checklist. You should use the mastery section to tick off and make sure that you have covered each topic. When you have revised the topic, you can tick the second column. Then try doing some questions – either from the textbook or the practice papers – and tick the final column once you feel confident with the topic.

The practice paper section shows the corresponding topic for each question in the textbook practice papers and the sets of practice papers in this book. You can use this to check the type of questions that you might get on each topic.

Practice papers and mark schemes

The best way to practise for exams is to do exams. These papers are designed to mimic the style of real IB papers. The questions often combine more than one syllabus topic and can require you to make novel links. As in the real exam papers, there is space for you to write in your calculations and answers to questions in Section A; for Section B, you will need to use a separate notebook.

Understanding mark schemes

Once you have tried some of the practice papers in this book, it is a good idea to mark your own (and also mark other people's) to see what makes things easy or difficult to award marks.

There are three different types of marks awarded in exams:

M These are method marks. They are awarded for a clear and obvious attempt to use the correct method. There is a certain amount of subjective opinion needed to award these. For example, if you are asked to find the length of the hypotenuse, h, of a right-angled triangle with shorter sides 5 and 7, which of the following would be awarded a method mark?

I $h = 5 + 7 = 12$

II $h = \sin(5) + \cos(7) = 1.08$

III $h = \sqrt{5 + 7} = 3.46$

IV $h = 5^2 + 7^2 = 74$

V $h = \sqrt{7^2 - 5^2} = 4.90$

VI $h = \sqrt{5^2 + 7^2} = 5 + 7 = 12$

Most examiners would agree that the first three examples are not good enough to award a method mark. In case VI, even though there is subsequent incorrect working and the wrong answer, a method mark would still be awarded. Cases IV and V are on the boundary of what might be acceptable and would probably require discussion among the examiners to find a clear boundary, but it is likely that both answers would be awarded a method mark. However, an answer of 74 or 4.90 by itself would not be awarded any marks because, even though we might have suspicions about where these numbers have come from, it has not been clearly communicated.

Sometimes method marks have brackets around them, for example, (**M1**). In this case they do not have to be explicitly seen and can be inferred from the correct answer.

Remember that sometimes the question requires a particular method (for example, find the maximum value of the function by differentiating) or it might require you to explicitly use the previous working (generally indicated by using the word 'hence'). If you use a different method in either of these instances, even if it works, you will not normally gain any credit.

For Paper 2, many questions will be answered primarily by using a calculator. However, you can still get some method marks for communicating what you are doing. Remember to write down any numbers that you put into your calculator that are not given in the question (for example, midpoints of grouped data). If you are using a graph to solve an equation, then draw a quick sketch.

A These are accuracy marks. They are for obtaining the correct answer. If there is a previous method mark without a bracket around it, then these marks can only be awarded if the previous method mark was awarded (this tends to happen in situations where examiners think the correct answer can be guessed and so they need to see supporting evidence, or when the question was a 'show that' or 'proof' question, where the communication rather than just the final answer is assessed).

Often lines are denoted **M1A1** – this means one method mark for a correct attempt and one accuracy mark for doing it correctly.

The accuracy mark is withheld if the value or expression given is wrong; however, it can also be withheld if the answer is written in calculator notation (for example, 1.8*E*9 rather than 1.8×10^9) or is given to the wrong accuracy – remember that all final answers should be either exact or written to three significant figures unless the question says otherwise. It is usually a good idea to write down intermediate working to more significant figures to ensure that the final answer is correct to at least three significant figures (and ideally store the answer to the full accuracy your calculator can hold using the calculator memory store).

Accuracy marks are also awarded when sketching graphs. It is important to choose an appropriate window to show all the important features of the graph and to label any relevant features (for example, axis intercepts, turning points, asymptotes).

Unless a particular form is required, most equivalent forms are accepted – for example, $x^2 + x$ or $x(x + 1)$ would normally both be fine. However, there is also an expectation that you understand the general requirements of the course. For example, if the question asked you to find the area under the curve $y = x^2$ between 0 and 1, the answer $\int_0^1 x^2 \mathrm{d}x$, while *technically* equivalent to the correct answer, is not sufficiently simplified – the acceptable answer would be $\frac{1}{3}$ or 0.333.

R These are marks awarded for clear reasoning – often in response to a question asking for explanation or justification. They might also be used when choosing particular solutions from equations (for example, saying that the solution of a quadratic that is negative cannot be a probability).

You may also see an **AG** notation in the mark schemes. This is used when the answer is given in the question. It is to remind the examiner that the correct answer does not mean that people have reasoned properly and to be particularly watchful for flawed arguments that just happen upon the right answer.

Sometimes later parts of a question need you to use an answer from a previous part. If you got the earlier part of the question wrong, the examiner will try to award 'follow through' marks by checking whether your method works for your prior incorrect answer. However, follow through marks may only be awarded if you have clearly communicated how you are using your previous answer, if you have not fundamentally changed the nature of the question, (for example, solving a quadratic equation turned into solving a linear equation) and if your answer is not implausible, (for example, a negative probability).

Revision tips

- Do not leave all your revision until the last minute. The IB is a two-year course with many later topics building on previous topics. One psychological study suggested that you need to 'learn' something seven times for it to really be fixed in your mind. Try to use each class test, mock exam or new topic as an opportunity to revise previous work.

- Revision should be an active rather than a passive process – often you can read through notes for hours and gain very little new knowledge. Try to do some questions first, then read through your notes and textbooks when you get stuck. Your reading will be far more focused if you are trying to find the solution to a particular difficulty.

- Try varied memorization strategies until you find one that works for you – copying out pages of notes does not work for most people. Strategies that do work for some people include using colour to prioritize key facts, using mind maps and making up silly songs to memorize techniques. Psychologists have found a strong link between memory and smell, so you could try using a particular perfume or deodorant while revising, then using the same one in the final exam!

- Work positively with others – some group revision can be a great way of improving your understanding as you can bounce ideas off each other, try to explain a concept to someone who is struggling or design exam-style questions for your friends to do. However, do be careful – avoid feeling bad by comparing yourself to people who seem to be good at everything and do not be tempted to feel good about yourself by making others feel bad – neither scenario is productive.

- Practise checking your answers. This is a vital skill that you will not suddenly be able to do in the final exam if you never do it in your revision. Think about good ways to check answers; for example, with and without a calculator, working backwards and sanity checking that the answer is plausible.

- Become an expert at using your exam calculator. You cannot start working on this skill too early, as each calculator has its own quirks that you need to get used to. Make sure you are using it in the mode required for the exam and know what happens when the memory is cleared and it is reset ahead of the exam; for example, does it default to radians or degrees mode?

- Become familiar with the exam formula booklet. It contains lots of useful information, but only if you are used to using it – make sure you know what all the symbols mean and where everything is, well ahead of your final exam. Formulae that are included in the formula booklet are indicated in the syllabus content sections of this book by this icon .

- Make sure some of your revision is under timed conditions. During the exam, the time flashes by for some people whereas others have to pace themselves or they run out of steam towards the end of an exam.

- Do not get downhearted if you are getting lots of things wrong, especially at the beginning of the revision process. This is absolutely normal – in fact, you learn a lot more from the things you get wrong than from the things you get right!

■ Weirdly, too much revision can be counterproductive. You will have your own personal concentration span beyond which there is no point in revising without a small break. Check that your revision plan is achievable, and schedule in plenty of relaxation time.

■ Try to get into stable sleeping and eating patterns in the run-up to the exam. If you are getting up each day at noon and never having caffeine, then a 9 a.m. exam with lots of coffee is unlikely to go well!

■ Unless you know that you only have a very good short-term memory, it is unlikely that the night before an exam is the best time to revise. Going for a run, doing some yoga or reading a good book and having a good night's sleep is likely to be worth far more marks than last minute panic revision.

■ If you choose to do any revision between Paper 1 and Paper 2, use the syllabus checklist to check if there are any major topics not covered in the first paper and focus your revision on those.

Exam tips

■ Use the reading time wisely. Every mathematics exam starts off with five minutes of reading time in which you are not allowed to write. This time is vital – make sure you read as much of the paper as you can and mentally start making a plan.

■ The examiners design the difficulty of the questions to be in increasing order in the short questions, and in increasing order within and between each long question; however, their judgment of difficulty is not always going to align with yours, so do not assume that you should do the questions in order. Many people try all the short questions first, spending too long on the last, often tricky, short question and then either panic or run out of time on the long questions. Sometimes the first long question is the easiest question on the paper, so consider doing that first. There is no substitute for potentially gaining lots of marks early on to build confidence for the rest of the exam.

■ Keep checking the time. Each mark equates to approximately one minute – so you do not spend 10 minutes on a question worth only 2 marks. Sometimes you have to abandon one question and move on to the next.

■ Do not get dispirited if you cannot do a question – the exam is meant to be challenging and you will not need 100% of the marks to get the grade you are aiming for. The worst thing you can do is let one bad question put you off showing your ability in other questions.

■ Look at the mark schemes to understand what is given credit. Even when many method marks are implied, only putting down the final answer is a high-risk strategy! Even the best mathematicians can make mistakes entering numbers into calculators. Mathematical communication is an important skill so try to convey your reasoning clearly – this has the advantage of enabling you to score some marks even if you make a mistake and of marshalling your ideas so you are more likely to get the answer right in the first place.

■ Especially in the long questions, do not assume that just because you cannot do an early part you cannot do later parts. If you get an early part wrong, follow through marks may still be available in later parts if you clearly communicate the correct method, even if you are using the wrong numbers. Sometimes the later parts of questions do not need the results from earlier parts anyway. The only way that you can guarantee getting no marks for part of a question is by leaving it blank!

■ In Paper 2, identify which questions are 'calculator questions'. Too many people try to do these questions using non-calculator techniques that do work, but often use up a lot of time.

■ Keeping the exam in perspective is perhaps more important than anything else. While it is of some importance, always remember that exams are artificial and imperfect measurements of ability. How much you can achieve working in silence, under timed conditions and by yourself without any resources on one particular set of questions is not what is most valued in mathematics. It should not be the only outcome from the course that matters, nor should it be how you judge yourself as a mathematician. It is only when you realize this that you will relax and have a chance of showing your true ability.

■ Finally, make sure that you understand the command terms used in exams – these are listed below. In particular, 'write down' means that you should be able to do it without any major work – if you find that your answer requires lots of writing, then you have missed something!

Command term	Definition
Calculate	Obtain a numerical answer showing the relevant stages in the working.
Comment	Give a judgment based on a given statement or result of a calculation.
Compare	Give an account of the similarities between two (or more) items or situations, referring to both (all) of them throughout.
Compare and contrast	Give an account of similarities and differences between two (or more) items or situations, referring to both (all) of them throughout.
Construct	Display information in a diagrammatic or logical form.
Contrast	Give an account of the differences between two (or more) items or situations, referring to both (all) of them throughout.
Deduce	Reach a conclusion from the information given.
Demonstrate	Make clear by reasoning or evidence, illustrating with examples or practical application.
Describe	Give a detailed account.
Determine	Obtain the only possible answer.
Differentiate	Obtain the derivative of a function.
Distinguish	Make clear the differences between two or more concepts or items.
Draw	Represent by means of a labelled, accurate diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Estimate	Obtain an approximate value.
Explain	Give a detailed account including reasons or causes.
Find	Obtain an answer showing relevant stages in the working.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.
Identify	Provide an answer from a number of possibilities.
Integrate	Obtain the integral of a function.
Interpret	Use knowledge and understanding to recognize trends and draw conclusions from given information.
Investigate	Observe, study, or make a detailed and systematic examination, in order to establish facts and reach new conclusions.
Justify	Give valid reasons or evidence to support an answer or conclusion.
Label	Add labels to a diagram.
List	Give a sequence of brief answers with no explanation.
Plot	Mark the position of points on a diagram.
Predict	Give an expected result.
Prove	Use a sequence of logical steps to obtain the required result in a formal way.
Show	Give the steps in a calculation or derivation.
Show that	Obtain the required result (possibly using information given) without the formality of proof. "Show that" questions do not generally require the use of a calculator.
Sketch	Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Solve	Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.
State	Give a specific name, value or other brief answer without explanation or calculation.
Suggest	Propose a solution, hypothesis or other possible answer.
Verify	Provide evidence that validates the result.
Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.

Calculator checklist

You should know how to:

	Skill	Got it!	Need to check
General	Change between radian and degrees mode		
	Set output to three significant figures		
	Store answers in calculator memory		
	Edit previous calculations		
Number and algebra	Input and interpret outputs in standard index form ($a \times 10^n$)		
	Use the sequence functions to find terms of an arithmetic and geometric sequence		
	Use tables to display sequences		
	Use the sum function to sum sequences		
	Use the TVM package to answer questions about compound interest and depreciation, including finding unknown interest rates and interest periods		
	Evaluate logarithms to any base		
Functions	Graph equations of the form $y = f(x)$		
	Use the zoom/window functions to explore interesting features of graphs		
	Use the trace function to explore graphs, especially suggesting asymptotes		
	Find the axis intercepts of graphs		
	Find the coordinates of local maxima or minima of graphs		
	Find the points of intersection of two graphs		
	Solve quadratic equations		
	Solve equations using solve functions on the calculator		
Probability and statistics	Input data, including from frequency tables and grouped data		
	Find mean, median, mode, quartiles and standard deviation from data		
	Input bivariate data		
	Find Pearson correlation coefficient of data		
	Find equation of y-on-x and x-on-y regression lines		
	Calculate probabilities for a given binomial distribution		
	Calculate probabilities for a given normal distribution		
	Calculate boundary values from probabilities for a given normal distribution		
	Calculate z-scores from probabilities of a normal distribution		
Calculus	Estimate the value of a limit from a table or a graph		
	Find the derivative of a function at a given point		
	Use the calculator to sketch the derivative of a function		
	Find definite integrals		
	Find areas/distances using definite integrals and the modulus (absolute value) function		

Syllabus revision

1 Number and algebra

■ Syllabus content

<table>
<tr><td rowspan="2">**S1.1**</td><td colspan="3">**Standard form**</td></tr>
<tr><td>Book Section 1B</td><td colspan="2">Revised ☐</td></tr>
<tr><td>*Syllabus wording*</td><td colspan="2">*You need to be able to:*</td><td>*Question*</td></tr>
<tr><td rowspan="3">Operations with numbers of the form
$a \times 10^k$ where $1 \leqslant a < 10$.</td><td colspan="2">Input and interpret numbers of this form on the calculator.</td><td>1 ☐</td></tr>
<tr><td colspan="2">Factorize to add or subtract numbers in standard form.</td><td>2 ☐</td></tr>
<tr><td colspan="2">Use the laws of exponents when multiplying or dividing numbers in standard form.</td><td>3 ☐</td></tr>
</table>

<table>
<tr><td rowspan="2">**S1.2**</td><td colspan="3">**Arithmetic sequences and series**</td></tr>
<tr><td>Book Section 2A</td><td colspan="2">Revised ☐</td></tr>
<tr><td>*Syllabus wording*</td><td colspan="2">*You need to be able to:*</td><td>*Question*</td></tr>
<tr><td rowspan="5">Use of the formulae for the *n*th term and the sum of the first *n* terms of the sequence.</td><td colspan="2">Find the *n*th term of an arithmetic sequence.
√x̄ $u_n = u_1 + (n-1)d$</td><td>4 ☐</td></tr>
<tr><td colspan="2">Use the formula to determine the number of terms in an arithmetic sequence.</td><td>5 ☐</td></tr>
<tr><td colspan="2">Set up simultaneous equations to find the first term and common difference.</td><td>6 ☐</td></tr>
<tr><td colspan="2">Find the sum of *n* terms of an arithmetic sequence. You should be able to use:
√x̄ $S_n = \dfrac{n}{2}(2u_1 + (n-1)d)$</td><td>7 ☐</td></tr>
<tr><td colspan="2">Or use:
√x̄ $S_n = \dfrac{n}{2}(u_1 + u_n)$</td><td>8 ☐</td></tr>
<tr><td rowspan="2">Use of sigma notation for sums of arithmetic sequences.</td><td colspan="2">Understand how sigma notation relates to arithmetic sequences.</td><td>9 ☐</td></tr>
<tr><td colspan="2">Evaluate expressions using sigma notation.</td><td>10 ☐</td></tr>
<tr><td rowspan="2">Applications.</td><td colspan="2">Recognize arithmetic sequences from descriptions.</td><td>11 ☐</td></tr>
<tr><td colspan="2">In particular, be aware that simple interest is a type of arithmetic sequence.</td><td>12 ☐</td></tr>
<tr><td>Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.</td><td colspan="2">Find the common difference as an average of the differences between terms.</td><td>13 ☐</td></tr>
</table>

<table>
<tr><td rowspan="2">**S1.3**</td><td colspan="3">**Geometric sequences and series**</td></tr>
<tr><td>Book Section 2B</td><td colspan="2">Revised ☐</td></tr>
<tr><td>*Syllabus wording*</td><td colspan="2">*You need to be able to:*</td><td>*Question*</td></tr>
<tr><td rowspan="4">Use of the formulae for the *n*th term and the sum of the first *n* terms of the sequence.</td><td colspan="2">Find the *n*th term of a geometric sequence.
√x̄ $u_n = u_1 r^{n-1}$</td><td>14 ☐</td></tr>
<tr><td colspan="2">Use the formula to determine the number of terms in a geometric sequence.</td><td>15 ☐</td></tr>
<tr><td colspan="2">Set up simultaneous equations to find the first term and common ratio.</td><td>16 ☐</td></tr>
<tr><td colspan="2">Find the sum of *n* terms of a geometric sequence using:
√x̄ $S_n = \dfrac{u_1(r^n - 1)}{r - 1} = \dfrac{u_1(1 - r^n)}{1 - r}, r \neq 1$</td><td>17 ☐</td></tr>
<tr><td rowspan="2">Use of sigma notation for sums of geometric sequences.</td><td colspan="2">Understand how sigma notation relates to geometric sequences.</td><td>18 ☐</td></tr>
<tr><td colspan="2">Evaluate expressions using sigma notation.</td><td>19 ☐</td></tr>
<tr><td>Applications.</td><td colspan="2">Recognize geometric sequences from descriptions.</td><td>20 ☐</td></tr>
</table>

Financial applications of geometric sequences		
S1.4 Book Section 2C	Revised ☐	
Syllabus wording	*You need to be able to:*	*Question*
Financial applications of geometric sequences and series. Compound interest Annual depreciation.	Calculate values of investments with compound interest using financial packages or the formula from the formula book: $$\text{\small ⓥ} \quad FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$$ Where: FV is the future value PV is the present value n is the number of years k is the number of compounding periods per year $r\%$ is the nominal annual rate of interest.	21 ☐
	Calculate interest rates required for particular outcomes.	22 ☐
	Calculate the number of periods required for a particular outcome.	23 ☐
	Calculate the values of goods suffering from depreciation.	24 ☐
	Calculate the real value of investments after inflation.	25 ☐

Exponents and logarithms		
S1.5 Book Section 1A, 1C	Revised ☐	
Syllabus wording	*You need to be able to:*	*Question*
Laws of exponents with integer exponents.	Evaluate expressions involving integer exponents including using $$a^m \times a^n = a^{m+n}$$ $$\frac{a^m}{a^n} = a^{m-n}$$ $$(a^m)^n = a^{mn}$$ $$a^{-n} = \frac{1}{a^n}$$ $$(ab)^n = a^n \times b^n$$ $$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$	26 ☐
	Simplify algebraic expressions using the above rules.	27 ☐
Introduction to logarithms with base 10 and e.	Use the fact that $\text{\small ⓥ} \quad a^x = b$ is equivalent to $\log_a b = x$	28 ☐
	Know that natural logarithms, $\ln x$, are equivalent to $\log_e x$ where e = 2.718 …	29 ☐
Numerical evaluation of logarithms using technology.	Use your calculator to evaluate logarithms to base 10 and e.	30 ☐

Proof		
S1.6 Book Section 11A	Revised ☐	
Syllabus wording	*You need to be able to:*	*Question*
Simple deductive proof, numerical and algebraic; how to lay out a LHS to RHS proof.	Write out LHS to RHS proofs.	31 ☐
The symbols and notation for equality and identity.	Know that the ≡ symbol is used to emphasize that a statement is true for all allowed values of a variable.	32 ☐

Further exponents and logarithms			
S1.7	Book Section 12A, 12B	Revised ☐	
Syllabus wording	*You need to be able to:*	*Question*	
Laws of exponents with rational exponents.	Work with rational exponents using $a^{\frac{1}{m}} = \sqrt[m]{a}$.	33	☐
Laws of logarithms.	Work with logarithms to bases other than 10 and e.	34	☐
	Manipulate logarithms algebraically using the laws given: $\log_a xy = \log_a x + \log_a y$ $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$	35	☐
Change of base of a logarithm.	Use $\log_a x = \dfrac{\log_b x}{\log_b a}$.	36	☐
Solving exponential equations.	Solve equations where the unknown is in the power by taking logarithms of both sides.	37	☐

Infinite geometric sequences			
S1.8	Book Section 13A	Revised ☐	
Syllabus wording	*You need to be able to:*	*Question*	
Sum of infinite convergent geometric sequences.	Use the formula: $S_\infty = \dfrac{u_1}{1 - r}$ to find the sum of an infinite geometric sequence.	38	☐
	Use the condition $|r| < 1$ to check if an infinite geometric sequence is convergent.	39	☐

Binomial expansions			
S1.9	Book Section 13B	Revised ☐	
Syllabus wording	*You need to be able to:*	*Question*	
The binomial theorem: expansion of $(a + b)^n$, $n \in \mathbb{N}$.	$(a + b)^n = a^n + {}^nC_1 a^{n-1} b + \dots {}^nC_r a^{n-r} b^r + \dots + b^n$	40	☐
	Approximate calculations using binomial expansions.	41	☐
	Work with single terms in binomial expansions.	42	☐
Use of Pascal's triangle and ${}^nC_r$	Evaluate binomial coefficients using either technology, Pascal's triangle or the formula: ${}^nC_r = \dfrac{n!}{r! \, (n - r)!}$	43	☐

Practice questions

1 Evaluate $(3 \times 10^{40})^2 - 5 \times 10^{80}$.

2 Evaluate $3 \times 10^{97} - 4 \times 10^{96}$, leaving your answer in the form $a \times 10^k$ where $1 \leqslant a < 10$.

3 Evaluate $(6 \times 10^{30}) \div (8 \times 10^{-12})$, leaving your answer in the form $a \times 10^k$ where $1 \leqslant a < 10$.

4 Find the 25th term of the following arithmetic sequence:
20, 17, 14, 11, …

5 An arithmetic sequence has first term 21 and last term 1602. If the common difference is 17, how many terms are in the sequence?

6 An arithmetic sequence has 4th term 10 and 10th term 34. Find the 20th term.

7 Find the sum of the first 30 terms of the arithmetic sequence 13, 10, 7, 4, …

8 An arithmetic sequence has $u_1 = 4$, $u_{20} = 130$. Find the sum of the first 20 terms.

9 Determine the first term and common difference of an arithmetic sequence where the sum of the first

terms is given by $S_n = \sum_{1}^{n}(5r + 11)$.

10 Evaluate $\sum_{1}^{100}(5r + 11)$.

11 On the first day of training for a race, Ahmed runs 500 m. On each subsequent day Ahmed runs 100 m further than the day before. How far has he run in total by the end of the 28th day?

12 Juanita invests $300 at 2.4% simple interest. How much will be in her account after 10 years?

13 A ball is dropped and the velocity (v m s^{-1}) is measured at different times (t seconds).

t	0	0.1	0.2	0.3
v	0	1.1	1.9	2.7

It is assumed that the velocity when $t = 0$ is correct, but there is uncertainty in the remaining measurements.

    a By modelling the situation as an arithmetic sequence, estimate the velocity when $t = 0.5$.

    b Make one criticism of the model.

14 Find the 10th term of the geometric sequence 32, –16, 8, –4, …

15 Find the number of terms in the geometric sequence 1, 2, 4, …, 4096.

16 A geometric series has third term 16 and seventh term 256. Find the possible values of the first term and the common ratio.

17 Find the sum of the first eight terms of the geometric sequence 162, 54, 18, …

18 Determine the first term and common ratio of a geometric sequence where the sum of the first

n terms is given by $S_n = \sum\limits_{1}^{n} 2 \times 5^r$.

19 Evaluate $\sum\limits_{1}^{10} 2 \times 5^r$.

20 The population of bacteria in a petri dish grows by 20% each day. There are initially 50 000 bacteria in the dish.

    a   Find the number of bacteria in the dish after 12 days.

    b   Explain why this model cannot continue indefinitely.

21 £2000 is invested in an account paying 4% nominal annual interest, compounded monthly. Find the amount in the account after 10 years, giving your answer to two decimal places.

22 Samira wants to invest £1000 in an account paying a nominal annual interest rate of i %, compounded quarterly. She wants to have £1500 in her account after 8 years. What value of i is required?

23 James invests $100 in an account paying 2.1% annual interest. How many complete years are required until he has doubled his investment?

24 A car suffers from 12% annual depreciation. If the initial value is $40 000, find the value after 4 years.

25 Clint invests $2000 at 3.2% annual compound interest. He estimates that the annual inflation rate is 2.4%. Find the real value of his investment after 5 years, giving your answer to the nearest dollar.

26 Evaluate $(2^{-2})^{-2}$.

27 Simplify $(2x)^3$.

28 Solve $4 \times 10^x = 5$, giving your answer in the form $x = \log_{10} a$.

29 If $e^{2x-6} = 5$, find x in terms of natural logarithms.

30 Evaluate $\ln 10 + \log_{10} e$.

31 Show that $\dfrac{1}{m-1} + \dfrac{1}{m+1} \equiv \dfrac{2}{m^2-1}$.

32 a If $x = ax + b$ find x in terms of a and b.

   b If $x \equiv ax + b$ for all x, find the values of a and b.

33 Evaluate $8^{\frac{2}{3}}$.

34 Evaluate $\log_4 32$.

35 If $x = \log_2 10$ and $y = \log_2 5$ write x in terms of y.

36 Write $\log_5 12$ in terms of natural logarithms.

37 Solve $5^{x-1} = 4 \times 3^{2x}$, giving your answer in the form $\frac{\ln a}{\ln b}$ where a and b are rational numbers.

38 Find the sum to infinity of the geometric sequence $2, \frac{2}{3}, \frac{2}{9}...$

39 Find the values of x for which the following geometric series is convergent: $1, 2x, 4x^2...$

40 Find the binomial expansion of $(2 + x)^4$.

41 Use your answer to question **40** to approximate 2.01^4, giving your answer to four significant figures.

42 Find the constant term in the expansion of $(2x - \frac{1}{x^3})^4$.

43 Evaluate $^8C_3$.

2 Functions

▪ Syllabus content

S2.1	**Equation of a straight line**		
	Book Section 4A	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Different forms of the equation of a straight line.	Use the gradient-intercept form $y = mx + c$, the general form $ax + by + d = 0$ and the point-gradient form $y - y_1 = m\,(x - x_1)$.	1	☐
	Find the equation of a line given its gradient and a point on the line.	2	☐
	Find the equation of a line given two points on the line using the gradient formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$	3	☐
Parallel lines $m_1 = m_2$.	Find the equation of a line through a given point parallel to another line.	4	☐
Perpendicular lines $m_1 \times m_2 = -1$.	Find the equation of a line through a given point perpendicular to another line.	5	☐

S2.2	**Concept of a function**		
	Book Section 3A	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Function notation.	Use function notation.	6	☐
Domain, range and graph.	Find the domain of a function.	7	☐
	Find the range of a function.	8	☐
Informal concept that an inverse function reverses or undoes the effect of a function.	Understand that an inverse function reverses the effect of a function.	9	☐
Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$.	Sketch the graph of the inverse function from the graph of the function.	10	☐

S2.3	**Graph of a function**		
	Book Section 3B	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Creating a sketch from information given or a context.	Sketch a graph from a list of features or from a given context.	11	☐
Using technology to graph functions.	Sketch the graph of a function from a plot on the GDC.	12	☐

S2.4	**Key features of graphs**		
	Book Section 3B	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Determine key features of graphs.	Use your GDC to find vertices (maximum and minimum values) and lines of symmetry.	13	☐
	Use your GDC to find vertical and horizontal asymptotes.	14	☐
	Use your GDC to find zeros of functions or roots of equations.	15	☐
Finding the point of intersection of two curves or lines using technology.	Use your GDC to find intersections of graphs.	16	☐

S2.5a	**Composite functions**		
	Book Section 14A	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Composite functions $(f \circ g)\,(x) = f(g(x))$.	Find the composite function of two functions.	17	☐
	Find the domain of a composite function.	18	☐

		Inverse functions	
S2.5b	Book Section 14B	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Finding the inverse function $f^{-1}(x)$.	Find the inverse of a function.		19 ☐
	Understand that a function has to be one-to-one to have an inverse.		20 ☐

		Quadratic functions	
S2.6	Book Section 15A	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
The quadratic function $f(x) = ax^2 + bx + c$: its graph, y-intercept $(0, c)$.	Identify the shape and y-intercept of the graph of a quadratic function from its equation. Know that if ⒜ $f(x) = ax^2 + bx + c$ then the axis of symmetry is $x = -\dfrac{b}{2a}$		21 ☐
The form $f(x) = a(x-p)(x-q)$: x-intercepts $(p, 0)$ and $(q, 0)$.	Identify the x-intercepts of the graph of a quadratic function by factorizing.		22 ☐
The form $f(x) = a(x-h)^2 + k$: vertex (h, k).	Identify the vertex and the line of symmetry of the graph of a quadratic function by completing the square.		23 ☐

		Quadratic equations and inequalities	
S2.7a	Book Section 15B	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Solution of quadratic equations and inequalities.	Solve quadratic equations by factorizing.		24 ☐
	Solve quadratic equations by completing the square.		25 ☐
The quadratic formula.	Solve quadratic equations by using the quadratic formula: ⒜ $ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$		26 ☐
	Solve quadratic inequalities.		27 ☐

		Quadratic discriminant	
S2.7b	Book Section 15C	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots.	Use the discriminant ⒜ $\Delta = b^2 - 4ac$ to determine whether a quadratic equation has two distinct real roots, one real root or no real roots.		28 ☐
	Use the discriminant to find the set of values of a parameter for which a quadratic equation has a given number of real roots.		29 ☐

		Rational functions	
S2.8	Book Section 16B	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
The reciprocal function $f(x) = \dfrac{1}{x}$, $x \neq 0$: its graph and self-inverse nature.	Sketch the graph of the reciprocal function $y = \dfrac{1}{x}$ and give the equations of the horizontal and vertical asymptotes.		30 ☐
Rational functions of the form $f(x) = \dfrac{ax + b}{cx + d}$ and their graphs.	Sketch the graph of rational functions of the form $y = \dfrac{ax + b}{cx + d}$ and give the equations of the horizontal and vertical asymptotes.		31 ☐

		Exponential and logarithmic functions	
S2.9	Book Section 16C	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Exponential functions and their graphs: $f(x) = a^x$, $a > 0$ $f(x) = e^x$.	Sketch the graph of exponential functions.		32 ☐
Logarithmic functions and their graphs: $f(x) = \log_a x$, $x > 0$; $f(x) = \ln x$, $x > 0$.	Sketch the graph of logarithmic functions.		33 ☐
	Understand the relationship between the graphs $y = a^x$ and $y = \log_a x$.		34 ☐
	⒜ Change an exponential from base a to base e using $a^x = e^{x \ln a}$.		35 ☐

S2.10a	Book Section 17A	**Solving equations analytically**

S2.10a	**Book Section 17A**	**Solving equations analytically**	
		Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Solving equations analytically.		Solve equations by factorizing.	36 ☐
		Solve disguised quadratics.	37 ☐
		Solve equations that lead to quadratics.	38 ☐

S2.10b	**Book Section 17B**	**Solving equations graphically**	
		Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach.		Use the graphing feature on your GDC to solve equations of the form $f(x) = 0$.	39 ☐
		Use the graphing feature on your GDC to solve equations of the form $f(x) = g(x)$.	40 ☐
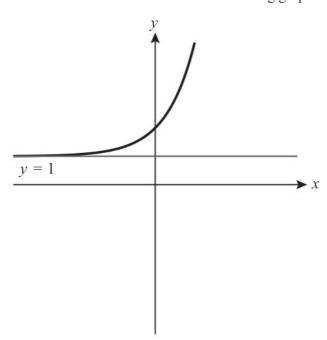
S2.11	**Book Section 16A**	**Transformations of graphs**	
		Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Translations: $y = f(x) + b$; $y = f(x - a)$.		Recognize that $y = f(x) + b$ is a vertical translation by b of $y = f(x)$.	41 ☐
		Recognize that $y = f(x - a)$ is a horizontal translation by a of $y = f(x)$.	42 ☐
Vertical stretch with scale factor p: $y = pf(x)$.		Recognize that $y = pf(x)$ is a vertical stretch with scale factor p of $y = f(x)$.	43 ☐
Horizontal stretch with scale factor $\frac{1}{q}$: $y = f(qx)$.		Recognize that $y = f(qx)$ is a vertical stretch with scale factor $\frac{1}{q}$ of $y = f(qx)$.	44 ☐
Reflections (in both axes): $y = -f(x)$; $y = f(-x)$.		Recognize that $y = -f(x)$ is a reflection in the x-axis of $y = f(x)$.	45 ☐
		Recognize that $y = f(-x)$ is a reflection in the y-axis of $y = f(x)$.	46 ☐
Composite transformations.		Apply two vertical transformations to a graph.	47 ☐
		Apply one horizontal and one vertical transformation to a graph.	48 ☐

■ Practice questions

1   Find the gradient and y-intercept of the line $3x - 4y - 5 = 0$.

2   Find the equation of the line with gradient -3 passing through the point $(2, -4)$. Give your answer in the form $y = mx + c$.

3   Find the equation of the line passing through the points $(-3, -5)$ and $(9, 1)$. Give your answer in the form $ax + by + d = 0$, where a, b, d are integers.

4   Find the equation of the line through the point $(1, 4)$ parallel to the line $y = 2x - 7$.

5   Find the equation of the line through the point $(-2, 3)$ perpendicular to the line $y = -\dfrac{1}{4}x + 1$.

6   If $f(x) = 3x^2 - 4$, find $f(-2)$.

7   Find the largest possible domain of the function $f(x) = \ln(2x - 1)$.

8   Find the range of the function $f(x) = \sqrt{(5 - x)}, x \leq 1$.

9   If $f(x) = 4 - 3x$, find $f^{-1}(-8)$.

10 Sketch the inverse function of the following graph:

11 The graph of $y = f(x)$ has zeros at -1 and 3 and no vertices. It has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = -2$.
The range of f is $f(x) > -2$.
Sketch a graph with these properties.

12 Sketch the graph of $y = x^5 - x^4 + 6x^2 - 2$, labelling the y-intercept.

13 a Find the coordinates of the vertices of $y = x^4 + 4x^3 - 3x^2 - 14x - 8$.

b Given that the curve has a line of symmetry, find its equation.

14 Find the equations of all vertical and horizontal asymptotes of the function $f(x) = \dfrac{x^2}{x^2 + x - 6}$.

15 Find the zeros of the function $f(x) = \dfrac{3}{\sqrt{x}} + 2x - 6$.

16 Find the points of intersection of $y = 3^x$ and $y = 3x + 2$.

17 $f(x) = \dfrac{1}{x - 2}$ and $g(x) = 3x - 4$.

Find

a $f(g(x))$

b $g(f(x))$.

18 $f(x) = \sqrt{2 - x}, x \leqslant 2$ and $g(x) = x - 3, x \in \mathbb{R}$.
Find the largest possible domain of $f(g(x))$.

19 $f(x) = \dfrac{x - 1}{x + 2}$

Find the inverse function $f^{-1}(x)$.

20 Find the largest possible domain of the function $f(x) = 2xe^x$, $x \geqslant k$ for which the inverse f^{-1} exists.
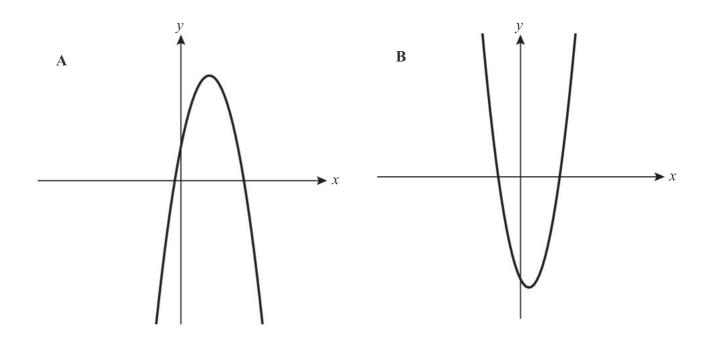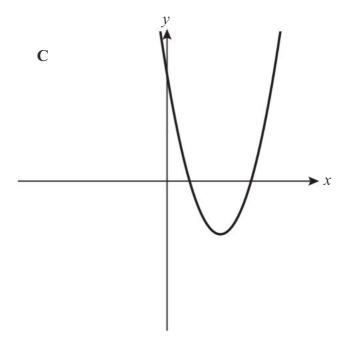
21 Match each equation to the corresponding graph.

    a $y = 2x^2 - 6x + 3$

    b $y = -2x^2 + 5x + 1$

    c $y = 4x^2 - 2x - 3$

A

B

C

22 Sketch the graph of $y = -3x^2 + 15x - 12$, labelling any axis intercepts.

23 a Express $x^2 - 4x + 7$ in the form $(x - h)^2 + k$.

b Hence, sketch the graph of $y = x^2 - 4x + 7$, labelling the coordinates of the vertex and any axis intercepts.

24 Solve, by factorizing, the equation $2x^2 + 7x - 15 = 0$.

25 a Express $x^2 - 5x + 3$ in the form $(x - h)^2 - k$.

b Hence solve the equation $x^2 - 5x + 3 = 0$.

26 Solve the equation $3x^2 - 4x - 2 = 0$.

27 Solve the inequality $x^2 - x - 12 > 0$.

28 By evaluating the discriminant, determine the number of distinct real roots of the equation $4x^2 + 5x + 3 = 0$.

29 Find the set of values of k for which the equation $3kx^2 + 4x + 12k = 0$ has two distinct real roots.

30 Sketch the graph of $y = \dfrac{1}{x}$, stating the equations of all asymptotes.

31 Sketch the graph of $y = \dfrac{3x - 2}{x + 1}$, labelling all axis intercepts and asymptotes.

32 Sketch the graph of $y = e^x$, labelling any axis intercepts.

33 Sketch the graph of $y = \log_2 x$, labelling any axis intercepts.

34 On the same axes, sketch the graphs of $y = 0.5^x$ and $y = \log_{0.5} x$, clearly showing the relationship between them.

35 Write 2.8^x in the form e^{kx}, giving the value of k to three significant figures.

36 Solve the equation $x \ln x = 4x$.

37 Solve the equation $x - 7\sqrt{x} + 10 = 0$.

38 Solve the equation $\log_2 x + \log_2(x + 2) = 3$.
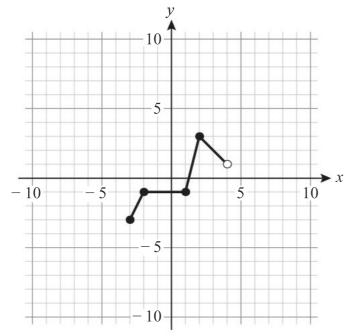
39 Solve the equation $x^2 e^x - 2x - 3 = 0$.
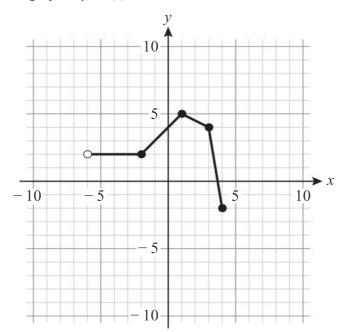
40 Solve the equation $2 \sin x = x^3 - x + 1$.

41 The graph of $y = f(x)$ is shown here.
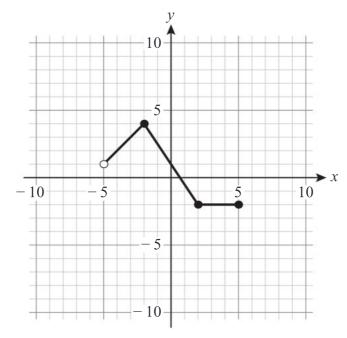
Sketch the graph of $y = f(x) - 4$.

42 The graph of $y = x^2 - 2x + 5$ is translated 3 units to the right. Find the equation of the resulting graph in the form $y = ax^2 + bx + c$.

43 The graph of $y = 3x^2 + x - 2$ is stretched vertically with scale factor 2. Find the equation of the resulting graph in the form $y = ax^2 + bx + c$.

44 The graph of $y = f(x)$ is shown here.

Sketch the graph of $y = f(2x)$.
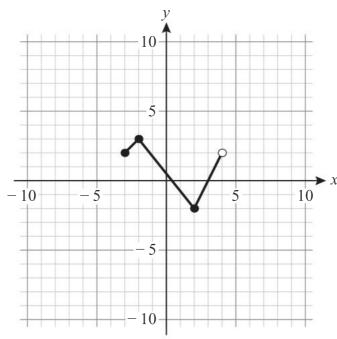
45 The graph of $y = f(x)$ is shown here.

Sketch the graph of $y = -f(x)$.

46 The graph of $y = x^3 + 3x^2 - 4x + 1$ is reflected in the y-axis.
Find the equation of the resulting graph in the form $y = ax^3 + bx^2 + cx + d$.

47 The graph of $y = f(x)$ has a single vertex at $(3, -2)$.
Find the coordinates of the vertex on the graph $y = 4f(x) + 1$.

48 The graph of $y = f(x)$ is shown here.

Sketch the graph of $y = 3f\left(\dfrac{1}{2}x\right)$.

3 Geometry and trigonometry

◼ Syllabus content

Distance and midpoints		
S3.1a	Book Section 4B	Revised ☐

Syllabus wording	*You need to be able to:*	*Question*
The distance between two points in three-dimensional space, and their midpoint.	Find the distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2) using $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$	1 ☐
	Find the midpoint using $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}, \dfrac{z_1 + z_2}{2} \right)$	2 ☐

Volume and surface area of 3D solids		
S3.1b	Book Section 5A	Revised ☐

Syllabus wording	*You need to be able to:*	*Question*
Volume and area of three-dimensional solids.	Find the volume and surface area of a sphere using $V = \dfrac{4}{3}\pi r^3$ $A = 4\pi r^2$ where r is the radius.	3 ☐
	Find the volume and curved surface area of a right cone using $V = \dfrac{1}{3}\pi r^2 h$ $A = \pi r l$ where r is the radius, h is the height and l is the slant height.	4 ☐
	Find the volume and surface area of a right pyramid using $V = \dfrac{1}{3}Ah$ where A is the area of the base and h is the height.	5 ☐
	Find the volume and surface area of combinations of solids.	6 ☐

Angle between intersecting lines and planes		
S3.1c	Book Section 5B & 5C	Revised ☐

Syllabus wording	*You need to be able to:*	*Question*
The size of an angle between two intersecting lines or between a line and a plane.	Find the angle between two lines in two dimensions.	7 ☐
	Find the angle between a line and a plane.	8 ☐
	Find the angle between two intersecting lines in three dimensions.	9 ☐

Trigonometry in right-angled triangles		
S3.2a	Book Section 5B	Revised ☐

Syllabus wording	*You need to be able to:*	*Question*
Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles.	Find lengths and angles in right-angled triangles using the sine, cosine and tangent ratios.	10 ☐

Trigonometry in non-right-angled triangles		
S3.2b	Book Section 5B	Revised ☐

Syllabus wording	*You need to be able to:*	*Question*
The sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$	Find lengths and angles in non-right-angled triangles using the sine rule.	11 ☐
The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$ $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$	Find lengths and angles in non-right-angled triangles using the cosine rule.	12 ☐
Area of a triangle as $\dfrac{1}{2}ab\sin C$.	Find the area of a triangle when you do not know the perpendicular height.	13 ☐

Applications of trigonometry			
S3.3 Book Section 5C	Revised ☐		
Syllabus wording	*You need to be able to:*		*Question*
Angles of elevation and depression.	Use trigonometry in questions involving angles of elevation and depression.	14	☐
Construction of labelled diagrams from written statements.	Construct diagrams from given information (often involving bearings) and solve using trigonometry.	15	☐

Radian measure and applications to circles			
S3.4 Book Section 18A	Revised ☐		
Syllabus wording	*You need to be able to:*		*Question*
The circle: radian measure of angles; length of an arc; area of a sector.	Convert between degrees and radians.	16	☐
	Find the length of an arc, $l = r\theta$, where r is the radius, θ is the angle measured in radians.	17	☐
	Find the area of a sector, $A = \dfrac{1}{2} r^2 \theta$ where r is the radius, θ is the angle measured in radians.	18	☐

Extending definitions of trigonometric functions			
S3.5 Book Section 18B	Revised ☐		
Syllabus wording	*You need to be able to:*		*Question*
Definition of $\cos\theta$, $\sin\theta$ in terms of the unit circle.	Use the definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle.	19	☐
Definition of $\tan\theta$ as $\dfrac{\sin\theta}{\cos\theta}$.	Use the definition $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.	20	☐
Exact values of trigonometric ratios of $0, \dfrac{\pi}{6}, \dfrac{\pi}{4}, \dfrac{\pi}{3}, \dfrac{\pi}{2}$ and their multiples.	Recall the exact values of sine, cosine and tangent of $0, \dfrac{\pi}{6}, \dfrac{\pi}{4}, \dfrac{\pi}{3}, \dfrac{\pi}{2}$ and their multiples.	21	☐
Extension of the sine rule to the ambiguous case.	Use the sine rule to find two possible solutions for an angle in a triangle.	22	☐

Trigonometric identities			
S3.6 Book Section 18C	Revised ☐		
Syllabus wording	*You need to be able to:*		*Question*
The Pythagorean identity $\cos^2\theta + \sin^2\theta \equiv 1$.	Use the identity $\cos^2\theta + \sin^2\theta \equiv 1$.	23	☐
Double angle identities for sine and cosine.	Use the identity $\sin 2\theta \equiv 2\sin\theta\cos\theta$.	24	☐
	Use the identities $\cos 2\theta \equiv \cos^2\theta - \sin^2\theta \equiv 2\cos^2\theta - 1 \equiv 1 - 2\sin^2\theta$.	25	☐

	Graphs of trigonometric functions		
S3.7	Book Section 18D	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
The circular functions $\sin x$, $\cos x$ and $\tan x$; amplitude, their periodic nature, and their graphs.	Sketch the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$.		26 ☐
Transformations.	Sketch the graphs of transformations of trigonometric functions.		27 ☐
Composite functions of the form $f(x) = a \sin (b(x + c)) + d$.	Sketch graphs of the form $y = a \sin(b(x + c))$ and $y = a \cos(b(x + c)) + d$.		28 ☐
Real-life contexts.	Form trigonometric models from given information.		29 ☐

	Solving trigonometric equations		
S3.8	Book Section 18E	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Solving trigonometric equations in a finite interval, both graphically and analytically.	Solve trigonometric equations graphically using your GDC.	30 ☐	
	Solve analytically trigonometric equations of the form $\sin \theta = k$, $\cos \theta = k$ and $\tan \theta = k$.	31 ☐	
	Solve analytically trigonometric equations of the form $A = k$, $\cos A = k$ and $\tan A = k$, where $A = f(\theta)$.	32 ☐	
	Use identities to solve trigonometric equations.	33 ☐	
Equations leading to quadratic equations in $\sin x$, $\cos x$ or $\tan x$.	Solve trigonometric equations as quadratics in a trigonometric function.	34 ☐	

■ Practice questions

1   Find the distance between (2, −4, 5) and (7, 3, −1).

2   Find the midpoint of (1, 8, −3) and (−5, 2, 4).

3   Find, to three significant figures, the volume and surface area of a sphere of diameter 16 cm.
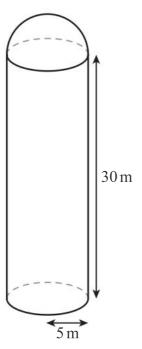
4   Find, to three significant figures, the volume and surface area of a cone with base radius 6 cm and height 15 cm.

5   Find, to three significant figures, the volume and surface area of a square-based pyramid with base side 5 cm and height 9 cm.

6   A grain silo is formed of a hemisphere on top of a cylinder of radius 5 m and height 30 m as shown:

30 m

5 m
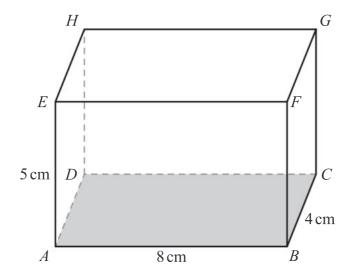
Find the silo's volume.

7 Find the acute angle between the lines $y = 4x - 3$ and $y = 5 - 3x$.
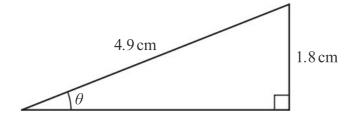
8 Find the angle between the line AG and the base plane $ABCD$ in the cuboid below.

9 Find the acute angle between the diagonals AG and EC in the cuboid from question **8**.
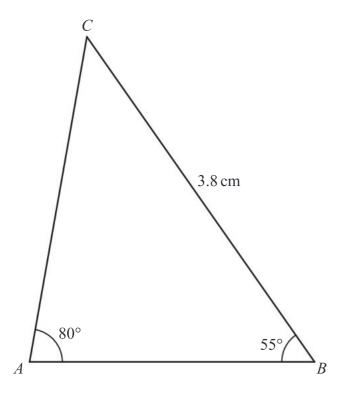
10 Find the angle θ in the following triangle.
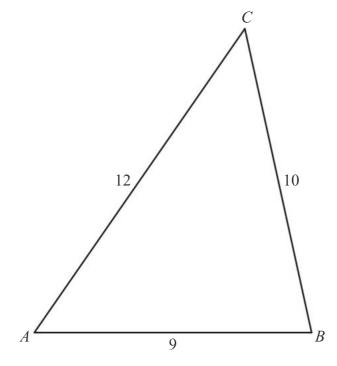
11 Find the length AC in the following triangle.
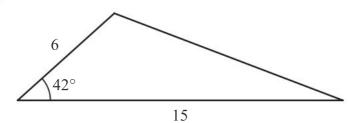
12 Find the angle C in the following triangle.

13 Find the area of the following triangle.

14 The angle of elevation of the top of a tree at a distance of 6.5 m is 68°.
Find the height of the tree.

15 A ship leaves port on a bearing of 030° and travels 150 km before docking. It then travels on a bearing of 110° for 80 km before docking again.
Find the distance it must now travel to return to where it started.
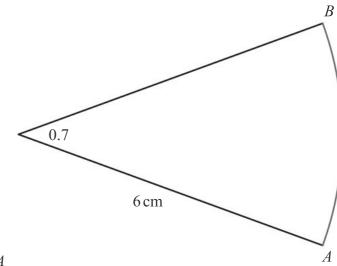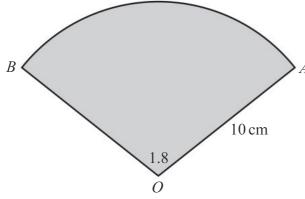
16 a Convert 55° to radians.

b Convert 1.2 radians to degrees.

17 Find the length of the arc AB.

18 Find the area of the sector AOB.

10 cm

1.8

O

0.7

6 cm

19 Given that $\sin \theta = 0.4$, find the value of
  a $\sin(\theta + \pi)$

  b $\cos(\theta - \frac{\pi}{2})$.
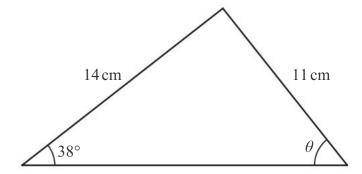
20 Using the definition of the tangent function, show that $\tan(2\pi - \theta) = -\tan \theta$.

21 Find the exact value of $\cos \frac{4\pi}{3}$.

22 Find the size of the angle θ in the following triangle.

23 Given that $\sin\theta = \dfrac{3}{4}$, where $\dfrac{\pi}{2} < \theta < \pi$, find the exact value of $\cos\theta$.

24 Prove that $(\cos\theta + \sin\theta)^2 \equiv \sin 2\theta + 1$.

25 Find the exact value of $\cos 15°$.

26 Sketch the graph of $y = \cos x$ for $-2\pi \leqslant x \leqslant 2\pi$, stating the period and amplitude.

27 Sketch the graph of $y = \tan\dfrac{x}{2}$ for $-360° \leqslant x \leqslant 360°$, labelling all axis intercepts and asymptotes.

28 Sketch the graph of $y = 3\sin\left(2\left(x - \dfrac{\pi}{3}\right)\right) + 5$ for $0 \leqslant x \leqslant 2\pi$.

29 A particle *P*, on the top of a spring which is fixed to a table, is initially at its maximum height above the table of 0.5 m. Its minimum height above the table, 0.16 m, occurs 0.3 s later.

Find a model of the form $h = a \cos bt + c$ for the height, *h* m, of *P* above the table at time *t* seconds.

30 Solve the equation $7\cos\left(2x - \dfrac{\pi}{5}\right) = 4$ for $0 < x < \pi$.

31 Solve the equation $\tan\theta = -\dfrac{\sqrt{3}}{3}$ for $-\pi < x < \pi$.

32 Solve the equation $\sin(x + 75°) = \dfrac{\sqrt{3}}{2}$ for $0° < x < 360°$.

33 Solve the equation $\sin 2\theta = \sin\theta$ for $0 \leqslant \theta \leqslant 2\pi$.

34 Solve the equation $2\sin^2 x - 3\cos x - 3 = 0$ for $-180° \leqslant x \leqslant 180°$.

4 Statistics and probability

■ Syllabus content

S4.1		**Sampling**	
	Book Section 6A	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Concepts of population, sample, random sample, discrete and continuous data.	Identify if data are continuous or discrete.		1 ☐
	Identify in context what the population is, what the sample is and whether it is random.		2 ☐
Reliability of data sources and bias in sampling.	Identify bias in sampling (a tendency for the sample to include more of one type of object).		3 ☐
	Identify reliability of data (strictly the consistency of their results and in a more colloquial sense, how trustworthy they are).		4 ☐
	Deal with missing data or errors in the recording of data.		5 ☐
Interpretation of outliers.	Know that an outlier is defined as more than $1.5 \times$ IQR from the nearest quartile, and be able to suggest how to determine if an outlier should be removed from the sample.		6 ☐
Sampling techniques and their effectiveness.	Be able to identify and evaluate the following sampling • techniques: • simple random • convenience • systematic • quota • stratified.		7 ☐
	Calculate the number of data items in each category of a stratified sample.		8 ☐

S4.2		**Statistical diagrams**	
	Book Section 6C	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Presentation of data: Frequency distributions.	Interpret frequency distribution tables.		9 ☐
Histograms.	Interpret frequency histograms.		10 ☐
Cumulative frequency graphs.	Interpret cumulative frequency graphs, including finding median, quartiles, percentiles, range and interquartile range using 🔘 $IQR = Q_3 - Q_1$		11 ☐
Box and whisker plots.	Produce box and whisker diagrams.		12 ☐
	Interpret box and whisker diagrams, including using them to compare distributions and use their symmetry to determine if a normal distribution is plausible.		13 ☐

S4.3		**Summary statistics**	
	Book Section 6B	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Measures of central tendency.	Calculate the mean, median and mode of data.		14 ☐
	Use the formula for the mean of data, 🔘 $\bar{x} = \dfrac{\sum_{i=1}^{k} f_i x_i}{n}$ where 🔘 $n = \sum_{i=1}^{k} f_i$		15 ☐
Estimation of mean from grouped data.	Use mid-interval values to estimate the mean of grouped data.		16 ☐
Modal class.	Find the modal class for grouped data using tables or histograms.		17 ☐
Measures of dispersion.	Use technology to calculate interquartile range (IQR), standard deviation and variance.		18 ☐
Effect of constant changes on the original data.	Calculate the mean and standard deviation (and other statistics) of the new data set after a constant change.		19 ☐
Quartiles of discrete data.	Use technology to obtain quartiles.		20 ☐

S4.4	**Correlation and regression**		
	Book Section 6D	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Linear correlation of bivariate data: Pearson's product-moment correlation coefficient, *r*.	Calculate the correlation coefficient of bivariate data using technology, and interpret the result, including being aware that correlation does not imply causation.		21 ☐
Scatter diagrams.	Estimate the line of best fit by eye, knowing that it should pass through the mean point.		22 ☐
Equation of the regression line of *y* on *x*.	Calculate the equation of the regression line using technology.		23 ☐
Use of the equation of the regression line for prediction purposes.	Use the regression line while being aware of the dangers of extrapolation. Be aware of when a *y* on *x* regression line is appropriate.		24 ☐
Interpret the meaning of the parameters, *a* and *b*, in a linear regression.	Put the meaning of the parameters into context.		25 ☐
Piecewise linear models.	Create and use piecewise linear models.		26 ☐

S4.5	**Definitions in probability**		
	Book Section 7A	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Concept of trial, outcome, equally likely outcomes, relative frequency, sample space and event.	Estimate probability from observed data.		27 ☐
The probability of an event *A* is $P(A) = \dfrac{n(A)}{n(U)}$	Find theoretical probabilities by listing all possibilities.		28 ☐
The complementary events *A* and *A'*.	Link the probability of an event occurring and it not occurring. $P(A) + P(A') = 1$		29 ☐
Expected number of occurrences.	Calculate how many times an outcome will be observed by multiplying the number of trials and the probability.		30 ☐

S4.6	**Probability techniques**		
	Book Section 7B	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities.	Use Venn diagrams to organize information and find probabilities.		31 ☐
	Use tree diagrams to organize information and find probabilities. In tree diagrams you multiply along the branches and add between the branches.		32 ☐
	Use sample space diagrams to organize information and find probabilities.		33 ☐
	Use tables of outcomes to organize information and find probabilities.		34 ☐
Combined events.	Work with the notation $A \cap B$ meaning *A* and *B* occurring. Work with the notation $A \cup B$ meaning *A* or *B* or both occurring. Use: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$		35 ☐
Mutually exclusive events.	Know that mutually exclusive means that the two events cannot both occur, so that $P(A \cap B) = 0$. Therefore $P(A \cup B) = P(A) + P(B)$		36 ☐
Conditional probability.	Know that $P(A|B)$ means the probability of *A* given that *B* has happened. Use Venn diagrams, tree diagrams, sample space diagrams or tables of outcomes to find conditional probabilities.		37 ☐
Independent events.	Know that if two events, *A* and *B*, are independent (do not affect each other) then $P(A \cap B) = P(A)P(B)$		38 ☐

Discrete random variables			
S4.7	Book Section 8A	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Concept of discrete random variables and their distribution.	Create probability distributions from context.		39 ☐
	Use the fact that the total probability in a probability distribution equals 1.		40 ☐
Expected value (mean) for discrete data.	Use: $$\text{E}(X) = \sum x\text{P}\,(X = x)$$		41 ☐
Applications.	Use probability distributions to answer questions in context.		42 ☐
	Know that $\text{E}(X) = 0$ indicates a fair game if X represents the gain of a player.		43 ☐

Binomial distribution			
S4.8	Book Section 8B	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Binomial distribution.	Recognize that if a situation has: • a fixed number of trials • outcomes that can be classified into two, 'successes' and 'failures' • fixed probability of being in each group • independent trials then the number of successes follow a binomial distribution.		44 ☐
	Use technology to calculate binomial probabilities.		45 ☐
Mean and variance of the binomial distribution.	Use: $$\text{E}(X) = np$$ $$\text{Var}(X) = np(1 - p)$$ where X is the number of successes when there are n binomial trials each with a probability p of success.		46 ☐

Normal distribution			
S4.9	Book Section 8C	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
The normal distribution and curve; properties of the normal distribution.	Recognize that many natural situations are well modelled by a normal distribution. One way to validate this is to use the fact that about 68% of normally distributed data should fall within one standard deviation of the mean, about 95% within two standard deviations and about 99.7% within three standard deviations.		47 ☐
Diagrammatic representation.	Recognize that a normal distribution can be represented by a symmetric bell-shaped curve with area representing probability.		48 ☐
Normal probability calculations.	For a given mean and standard deviation, find the probability of a random variable falling in a given interval.		49 ☐
Inverse normal calculations.	For a given probability, find the boundary of the region it describes.		50 ☐

***x* on *y* regression**			
S4.10	Book Section 19A	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Equation of the regression line of x on y.	Use your GDC to find the regression line of x on y for a given data set.		51 ☐
Use of the equation for prediction purposes.	Use the regression line to predict values of x for given values of y.		52 ☐

S4.11		**Formal conditional probability**	
	Book Section 19B	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Formal definition and use of the formulae: $P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$ for conditional probabilities, and $P(A\|B) = P(A)$ for independent events.	Find conditional probabilities using the formula $\quad P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$		53 ☐
	Test whether two events are independent.		54 ☐

S4.12		**Standardizing normal variables**	
	Book Section 19C	Revised ☐	
Syllabus wording	*You need to be able to:*		*Question*
Standardization of normal variables (*z*-values).	Find *z*-values and know that they give the number of standard deviations from the mean. Use the definition $\quad z = \dfrac{x - \mu}{\sigma}$		55 ☐
Inverse normal calculations where mean and standard deviation are unknown.	Use the inverse normal distribution on your GDC and *z*-values to find an unknown mean and standard deviation.		56 ☐

▬ Practice questions

1   Determine whether each of the following variables is continuous or discrete.
   a   Number of people in a family.
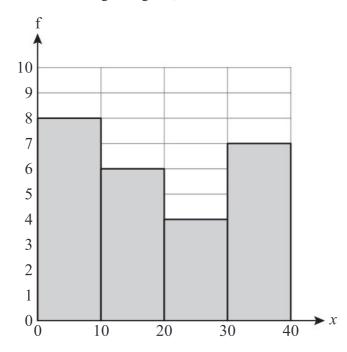
   b   Time for a nucleus to decay.

   c   Age in complete years.

2   A doctor wants to find out whether exercise can lower the incidence of illness. He asks patients who come to his clinic to fill in a survey about their exercise habits. 20% of them agree to do this.
   a   Suggest a possible population that the doctor is interested in.

   b   Is his sample random?

3   Is the sampling in question **2** likely to be biased? Justify your answer.

4   Five independent groups of people were asked to estimate the length of an arrow which is 5 cm long. The average for the groups was 4.6 cm, 4.6 cm, 4.7 cm, 4.8 cm, 4.8 cm. Does this suggest that the results are reliable?

5   Five people were asked to record their height in metres:
   A: 1.83   B: 1.45   C: 1.77   D: 5.10   E: 1.60
   Suggest which data item is an error. What should be done with this item?

6   A data set has lower quartile 7 and upper quartile 11. Explain why 18 should be considered an outlier and suggest how to determine if it should be excluded from the data.

7 Write down the sampling method used by the doctor in question **2**.

8 A language school consists of students from either Italy or Spain. There are 60 from Italy and 90 from Spain. In a stratified sample of 20 students, how many should be from Italy?

9 For the following frequency table, find the proportion of data items above 20.
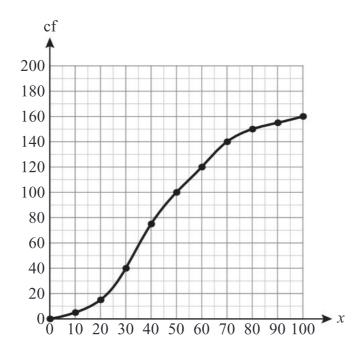
x	$0 < x \leqslant 20$	$20 < x \leqslant 30$	$30 < x \leqslant 40$
Frequency	15	18	12

10 For the following histogram, estimate the number of data items above 25.
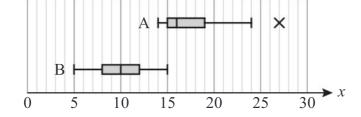
11 For the following cumulative frequency graph find

   a   the median

   b   the interquartile range

   c   the 90th percentile.

12 Sketch a box and whisker plot for the sample below:
12, 13, 15, 16, 16, 18, 18, 19, 20.

13 For the following box and whisker plots:

   a   Compare the two distributions.

   b   Determine, with justification, which of the two distributions is more likely to be a normal distribution.

14 Write down the mean, median and mode of the following data:
14, 14, 16, 17, 19, 20, 23, 25

15 The numbers 4, 8, 2, 9, and x have a mean of 7.
Find the value of x.

16 a Estimate the mean of the following grouped data.

x	$10 < x \leqslant 20$	$20 < x \leqslant 30$	$30 < x \leqslant 50$	$50 < x \leqslant 60$
Frequency	10	12	15	13

b Explain why it is only an estimate.

17 Find the modal class for the data below:

x	$0 < x \leqslant 5$	$5 < x \leqslant 10$	$10 < x \leqslant 15$	$15 < x \leqslant 20$
Frequency	16	12	15	18
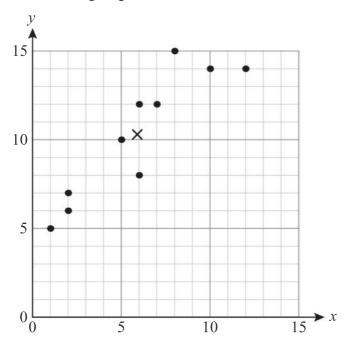
18 For the data set 6, 7, 9, 12, 14, 18, 22 find
a the interquartile range

b the standard deviation

c the variance.

19 A set of data has mean 12 and standard deviation 10. Every item in the data set is doubled, then 4 is added on. Find the mean and standard deviation of the new data set.

20 Find the quartiles of the following data:

17, 15, 23, 29, 15, 22, 28, 30.

21 a Calculate Pearson's product-moment correlation coefficient for the following data:

x	2	4	4	7
y	1	3	6	8

b Interpret your result.

22 The following diagram shows a set of 10 data items with the mean point labelled with a cross.

a Sketch on a line of best fit.

b Hence estimate y when $x = 3$.

23 Find the y-on-x regression line for the following data, in which y is dependent on x.

x	1	2	2	5	6	6	7	8	10	12
y	5	6	7	10	12	8	12	15	14	14

24 a Use your answer to question **23** to estimate:
    i y when $x = 9$

    ii y when $x = 20$

    iii x when $y = 10$.

  b Which of the predictions made in part **a** is valid. Justify your answer.

25 A social scientist investigates how the number of text messages sent by pupils each day (y) depends on the number of hours they spend on social media each day (x). He finds the regression line $y = 6.7 + 1.4x$.

Interpret what each of the following numbers mean in context.
a 6.7
b 1.4

26 A veterinary researcher believes that the growth of a breed of snake is very different during the first six months compared to the next six months.
She collects the following data showing the length (L cm) and age (A months) of a sample of snakes.

A	1	2	4	4	7	7	10	11	12
L	4	8	15	18	30	32	34	36	34

a Create a piecewise linear model to reflect the researcher's belief.
b Use your answer to part **a** to estimate the length of a 3-month-old snake of this breed.

27 A coin is flipped 200 times and 134 heads are observed. Estimate the probability of observing a head when the coin is flipped.

28 Find the probability of rolling a prime number on a fair six-sided dice.

29 If P(A) = 0.6, find P(A').

30 If there are 30 pupils in a class and the probability of a student being absent is 0.05, find the expected number of absent pupils.

31 In a class of 30 students, 14 study French, 18 study Spanish and 4 study both languages. Find the probability that a randomly chosen student studies neither French nor Spanish.

32 A drawer contains three white socks and five black socks. Two socks are drawn without replacement.
   a   Find P(2nd sock is black|1st sock is white).

   b   Find the probability that the socks are different colours.

33 A fair four-sided dice is thrown twice.
   a   What is the probability that the total score is greater than 5?

   b   If the total score is greater than 5, what is the probability that it is 7?

34 100 students were asked whether they preferred soccer or cricket. They were also asked if they prefer mathematics or art. The results are summarized below:

	Soccer	**Cricket**
Mathematics	40	20
Art	30	x

   a   Find the value of x.

   b   Find the probability that a randomly chosen student prefers mathematics to art.

35 If P(A) = 0.5, P(B) = 0.7 and P($A \cap B$) = 0.3 find P($A \cup B$).

36 Events *A* and *B* are mutually exclusive. If P(*A*) = 0.4 and P(*B*) = 0.2, find P(*A* ∪ *B*).

37 For the sample in question **34**, determine the probability that a randomly chosen person who prefers soccer also prefers mathematics.

38 Independent events *A* and *B* are such that P(*A*) = 0.4 and P(*B*) = 0.6. Find P(*A* ∩ *B*).

39 A drawer contains three white socks and four black socks. Two socks are drawn at random without replacement. Find the probability distribution of *W*, the number of white socks drawn.

40 The random variable *X* can take values 0, 1 or 2 with probability P(*X* = *x*) = *k* (*x* + 1). Find the value of *k*.

41 For the distribution given below, find E(*X*).

x	0.5	1	2.5
P(*X* = *x*)	0.5	0.4	0.1

42 The value of prizes ($*X*) won by an individual each month in a prize draw is shown below.

X	0	10	2000
P(*X* = *x*)	0.9	0.095	0.005

a Given that an individual wins a prize, find the probability that it is $2000.
b Find the probability of winning more than the expected amount.

43 The gain, $X, of a player in a game of chance follows the distribution shown in the table.

X	-1	0	k
P($X = x$)	0.6	0.3	0.1

Find the value of k that would make the game fair.
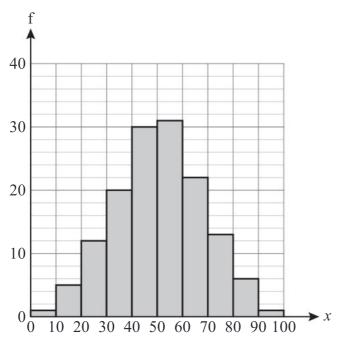
44 A drawer contains 5 black socks and 10 red socks. Four socks are drawn at random without replacement. Explain why the number of black socks drawn does not follow a binomial distribution.

45 If X is a random variable following a binomial distribution with five trials and a probability of success of 0.4, find
  a P($X = 2$)

  b P($X \geqslant 3$).

46 A biased coin has a probability of 0.6 of showing a head. It is tossed 10 times.
  If this experiment is repeated many times:
  a Find the expected mean number of heads.

  b Find the expected standard deviation in the number of heads.

47 The time for a child to learn a new dance move is found to have a mean of 2 weeks and a standard deviation of 4 weeks. Explain why this variable is unlikely to be modelled by a normal distribution.

48 The following histogram shows the results of an experiment.

a What feature of this graph suggests a normal distribution might be a good model for the outcome of the experiment?

b Visually estimate the mean of the distribution.

49 A random normal variable has mean 12 and standard deviation 2. Find the probability that an observation is between 11 and 15.

50 A random normal variable has mean 100 and standard deviation 15. The probability of being above k is 0.7. Find the value of k.

51 Find the equation of the regression line of x on y for the following data:

x	1	2	3	3	4	5	7	8
y	18	20	17	15	16	12	10	11

52 Based on data with y-values between 15 and 30 and a correlation coefficient of 0.89, the following regression line is found:

$$x = 1.82y - 11.5$$

    a Use this line to predict the value of x when
       i $y = 20$

       ii $y = 35$.

    b Comment on the reliability of your predictions in each case.

53 $P(A) = 0.6$ and $P(A \cap B) = 0.4$.
    Find $P(B|A)$.

54 $P(A) = 0.2$, $P(B) = 0.8$ and $P(A \cup B) = 0.7$.
    Determine whether A and B are independent.

55 Given that $X \sim N(10, 4.8^2)$, find how many standard deviations $x = 17$ is from the mean.

56 $X \sim N(\mu, \sigma^2)$ with $P(X < 12) = 0.3$ and $P(X > 34) = 0.2$.
    Find μ and σ.

5 Calculus

◼ Syllabus content

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<tr>
<td colspan="2" rowspan="2">Introduction to the concept of a limit.</td>
<td colspan="2">Estimate the value of a limit from a table.</td>
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<td colspan="2">Estimate the value of a limit from a graph.</td>
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<td colspan="2" rowspan="4">Derivative interpreted as gradient function and as rate of change.</td>
<td colspan="2">Understand and use the notation for derivatives: $\frac{dy}{dx}$ and f$'(x)$.</td>
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<tr>
<td colspan="2">Interpret the derivative as a rate of change.</td>
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<td rowspan="3">Graphical interpretation of f$'(x) > 0$, f$'(x) = 0$, f$'(x) < 0$.</td>
<td colspan="2">Identify intervals on which a function is increasing (f$'(x) > 0$) and decreasing (f$'(x) < 0$).</td>
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<td colspan="2">Sketch the graph of the derivative from the graph of a function.</td>
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<tr>
<td colspan="2">Sketch the graph of a function from the graph of its derivative.</td>
<td>9 ☐</td>
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<td rowspan="2">S5.3</td>
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<td>Syllabus wording</td>
<td colspan="2">You need to be able to:</td>
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<tr>
<td rowspan="2">The derivative of functions of the form
$f(x) = ax^n + bx^{n-1} + \cdots$
where all exponents are integers.</td>
<td colspan="2">Apply the rule to differentiate polynomials using
⒡   f$(x) = x^n \Rightarrow$ f$'(x) = nx^{n-1}$</td>
<td>10 ☐</td>
</tr>
<tr>
<td colspan="2">Rearrange an expression into the form f$(x) = ax^n + bx^{n-1} + \cdots$ before differentiating.</td>
<td>11 ☐</td>
</tr>
</table>

<table>
<tr>
<td rowspan="2">S5.4</td>
<td colspan="3" align="center">Equations of tangents and normals</td>
</tr>
<tr>
<td>Book Section 9D</td>
<td colspan="2">Revised ☐</td>
</tr>
<tr>
<td>Syllabus wording</td>
<td colspan="2">You need to be able to:</td>
<td>Question</td>
</tr>
<tr>
<td rowspan="7">Tangents and normals at a given point, and their equations.</td>
<td colspan="2">Evaluate the gradient at a given point.</td>
<td>12 ☐</td>
</tr>
<tr>
<td colspan="2">Find the point on the curve with a given gradient.</td>
<td>13 ☐</td>
</tr>
<tr>
<td colspan="2">Find the equation of the tangent to the curve $y =$ f(x) at the point (x_1, y_1) using
$$y - y_1 = m(x - x_1)$$
where $y_1 =$ f(x_1) and $m =$ f$'(x_1)$.</td>
<td>14 ☐</td>
</tr>
<tr>
<td colspan="2">Find the equation of the normal to the curve using
$$y - y_1 = -\frac{1}{m}(x - x_1)$$</td>
<td>15 ☐</td>
</tr>
<tr>
<td colspan="2">Solve problems involving tangents and normals.</td>
<td>16 ☐</td>
</tr>
<tr>
<td colspan="2">Use technology to find the gradient and the equation of the tangent at a given point.</td>
<td>17 ☐</td>
</tr>
<tr>
<td colspan="2">Use technology to draw the graph of the gradient function.</td>
<td>18 ☐</td>
</tr>
</table>

		Introduction to integration		
S5.5	Book Section 10A, 10B	Revised ☐		
Syllabus wording		*You need to be able to:*		*Question*
Integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + \cdots$, where $n \in \mathbb{Z}$, $n \neq -1$.		⒜ Use $\int ax^n dx = \dfrac{a}{n+1}x^{n+1} + c$, for $n \neq -1$.		19 ☐
		Rearrange an expression into the form $f(x) = ax^n + bx^{n-1} + \ldots$ before integrating.		20 ☐
Anti-differentiation with a boundary condition to determine the constant term.		Find the expression for y in terms of x when given $\dfrac{dy}{dx}$ and one pair of (x, y) values.		21 ☐
Definite integrals using technology. Area of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x) > 0$.		⒜ Use technology to evaluate integrals of the form $\int_a^b f(x)\, dx$, and interpret this as the area between the curve and the x-axis.		22 ☐

		Further differentiation		
S5.6	Book Section 20A, 20B, 20C	Revised ☐		
Syllabus wording		*You need to be able to:*		*Question*
Derivative of x^n ($n \in \mathbb{Q}$), $\sin x$, $\cos x$, e^x and $\ln x$. Differentiation of a sum and a multiple of these functions.		Apply the rules of differentiation to these functions. ⒜ $f(x) = \sin x \Rightarrow f'(x) = \cos x$ ⒜ $f(x) = \cos x \Rightarrow f'(x) = -\sin x$ ⒜ $f(x) = e^x \Rightarrow f'(x) = e^x$ ⒜ $f(x) = \ln x \Rightarrow f'(x) = \dfrac{1}{x}$		23 ☐
		Evaluate the gradient at a given point.		24 ☐
The chain rule for composite functions.		⒜ $y = g(u)$ where $u = f(x) \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$		25 ☐
The product and quotient rules.		⒜ $y = uv \Rightarrow \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$		26 ☐
		⒜ $y = \dfrac{u}{v} \Rightarrow \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$		27 ☐

		Second derivative		
S5.7	Book Section 20D	Revised ☐		
Syllabus wording		*You need to be able to:*		*Question*
The second derivative.		Find the second derivative and understand the notation $f''(x)$ and $\dfrac{d^2y}{dx^2}$.		28 ☐
Graphical behaviour of functions, including the relationship between the graphs of f, f′ and f″.		Sketch the graph of $y = f''(x)$ given the graph of $y = f(x)$.		29 ☐
		Describe sections of a graph as 'concave up' or 'concave down'.		30 ☐
		Use the second derivative to determine whether a graph is concave up ($f''(x) > 0$) or concave down ($f''(x) < 0$).		31 ☐

		Maximum, minimum and inflection points		
S5.8	Book Section 20E, 20F	Revised ☐		
Syllabus wording		*You need to be able to:*		*Question*
Local maximum and minimum points. Testing for maximum and minimum.		Locate local maximum and minimum points by solving $f'(x) = 0$.		32 ☐
		Use the second derivative to distinguish between maximum ($f''(x) < 0$) and minimum ($f''(x) > 0$).		33 ☐
Optimization.		Find the maximum or minimum value of a function in a real-life context.		34 ☐
Points of inflection with zero and non-zero gradients.		Locate points of inflection by solving $f''(x) = 0$ and checking that the concavity of the function changes at that point.		35 ☐

		Introduction to kinematics		
S5.9	Book Section 21C	Revised ☐		
Syllabus wording		*You need to be able to:*		*Question*
Kinematic problems involving displacement *s*, velocity *v*, acceleration *a* and total distance travelled.		Use differentiation to find velocity and acceleration. $\quad a = \dfrac{dv}{dt} = \dfrac{d^2 s}{dt^2}$	36	☐
		Find speed as the magnitude of velocity.	37	☐
		Integrate velocity to find the displacement. displacement $= \displaystyle\int_{t_1}^{t_2} v(t)\,dt$	38	☐
		Integrate speed to find distance travelled. distance $= \displaystyle\int_{t_1}^{t_2} |v(t)|\,dt$	39	☐

		Further integration techniques		
S5.10	Book Section 21A	Revised ☐		
Syllabus wording		*You need to be able to:*		*Question*
Indefinite integral of x^n ($n \in \mathbb{Q}$), $\sin x$, $\cos x$, $\dfrac{1}{x}$ and e^x.		Apply integration rules to these functions. $\displaystyle\int \dfrac{1}{x}\,dx = \ln|x| + c$ $\displaystyle\int \sin x\,dx = -\cos x + c$ $\displaystyle\int \cos x\,dx = \sin x + c$ $\displaystyle\int e^x\,dx = e^x + c$	40	☐
The composites of any of these with the linear function $ax + b$.		When integrating $f(ax + b)$, remember to divide by a.	41	☐
Integration by inspection (reverse chain rule) or by substitution for expressions of the form: $\displaystyle\int k g'(x) f(g(x))\,dx$		Use reverse chain rule for integrals where a composite function is multiplied by the derivative of the inner function: $\displaystyle\int g'(x) f(g(x))\,dx = f(g(x)) + c$	42	☐

		Evaluating definite integrals		
S5.11	Book Section 21B	Revised ☐		
Syllabus wording		*You need to be able to:*		*Question*
Definite integrals, including the analytical approach.		Evaluate definite integrals by using $\displaystyle\int_a^b g'(x)\,dx = g(b) - g(a)$	43	☐
		Recognize that some definite integrals can only be found using technology.	44	☐
Areas of a region enclosed by a curve $y = f(x)$ and the *x*-axis, where $f(x)$ can be positive or negative, without the use of technology.		Use definite integrals to find areas without a calculator using $\quad A = \displaystyle\int_a^b y\,dx$	45	☐
		Split the area into positive and negative parts before calculating the definite integrals.	46	☐
Areas between curves.		Find the area between two curves by using $\displaystyle\int_a^b (f_1(x) - f_2(x))\,dx$ You may need to find the intersection points first.	47	☐

■ Practice questions

1 In this question, x is measured in degrees. Use a table to estimate, to two decimal places, the limit of $\dfrac{\sin 3x}{0.2x}$ when x tends to zero.

2 Use a graph to estimate the limit of $\dfrac{\ln\left(\frac{x}{2}\right)}{x-2}$ when x tends to 2.

3 Given that $y = 3x^2 - 5x$ and $\dfrac{dy}{dx} = 6x - 5$, what is the value of the derivative of y when $x = 2$?

4 Write an equation to represent the following situation:
   The area decreases with time at a rate proportional to the current area.

5 The table shows some information about a function f(x).

x	1	3	4
f(x)	4	8	5
f′(x)	−1	4	2
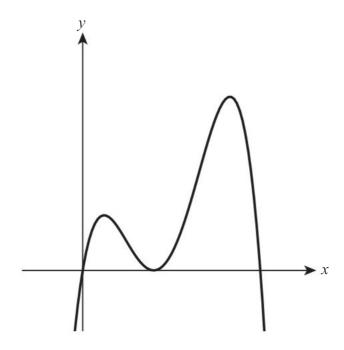
   A graph has equation $y = $ f(x). Find the gradient of the graph at the point where $y = 4$.

6 Point $P(4,2)$ lies on the curve with equation $y = \sqrt{x}$. The table shows the coordinates of a variable point Q and the gradient of the chord PQ.
   Complete the table and use it to estimate the gradient of the curve at P.
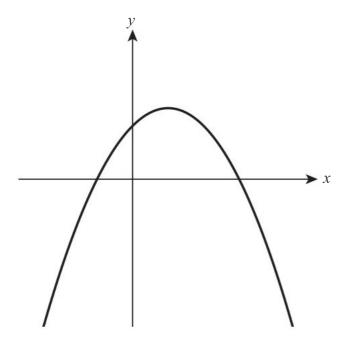
x_Q	y_Q	Δx	Δy	Gradient of PQ
5	2.236	1	0.236	0.236
4.1	2.025	0.1		
4.01				
4.001				

7   Use technology to sketch the graph of $f(x) = x^3 - 5x + 2$ and to find the range of values of x for which $f'(x) < 0$.

8   The graph of $y = f(x)$ is shown below. Sketch the graph of $y = f'(x)$.

9   The graph of $y = f'(x)$ is shown below. Sketch one possible graph of $y = f(x)$.

10 Differentiate $y = 4x^2 - \dfrac{1}{10}x^{-5} - 3x + 2$.

11 Find f′(x) when

    a   f$(x) = 3x^2(4 - x^4)$

    b   f$(x) = 1 - \dfrac{3}{2x^4}$

    c   f$(x) = \dfrac{4x^2 - 3x + 1}{5x}$

12 Given that f$(x) = 4x^2 - 2x^{-1}$, evaluate f′(2).

13 Find the x-coordinates of the points on the curve $y = 12x + 5x^{-1}$ where the gradient equals 2.

14 A curve has equation $y = x^2 - 3$.
    Find the equation of the tangent to the curve at the point where $x = 4$.

15 Find the equation of the normal to the curve $y = 3x - 2x^{-1}$ at the point where $x = 2$.

16 The tangent to the curve with equation $y = x^2 - 3$ at the point (a, b) passes through $(0, -12)$.
Find the possible values of a.

17 A curve has equation $y = \dfrac{4\sqrt{x} - 3}{7x^2}$.

Find, correct to two decimal places:
a the gradient when $x = 3.2$

b the equation of the tangent at the point where $x = 3.2$.

18 A curve has equation $y = \dfrac{4\sqrt{x} - 3}{7x^2}$.
Find the coordinates of the point on the curve where the gradient is 2.
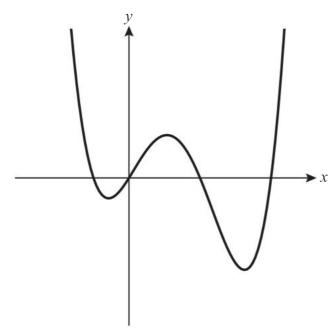
19 Find $\displaystyle\int 9x^2 + 6x^{-3}\,dx$.

20 Find $\displaystyle\int \dfrac{x^5 - 3}{2x^2}\,dx$.

21 Given that $\dfrac{dy}{dx} = 4x + 2$, and that $y = 3$ when $x = 2$, find an expression for y in terms of x.

22 Find the area enclosed by the curve $y = 2x^3 - 1$, the x-axis and the lines $x = 2$ and $x = 3$.
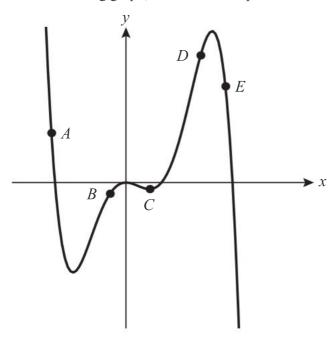
23 Differentiate $3\sin x - 5\cos x + 2$.

24 Given that $f(x) = 2\sqrt{x} - 3\ln x$, evaluate $f'(9)$.

25 Differentiate:

   a $\sqrt{3x^2 - 1}$

   b $2\sin^3(5x)$.

26 Given that $y = 4x e^{-3x}$, find $\dfrac{dy}{dx}$.

27 Given that $f(x) = \dfrac{\ln x}{4x}$, show that $f'(x) = \dfrac{1 - \ln x}{4x^2}$.

28 Given that $y = x^3 - 3 \ln x$, find $\dfrac{d^2y}{dx^2}$.

29 The diagram shows the graph of $y = f(x)$. Sketch the graph of $y = f''(x)$.

30 For the following graph, write down the points at which the function is concave-down.

31 Find the range of values of x for which the function $f(x) = 5x^3 - 2x^2 + 1$ is concave-up.

32 The curve with equation $y = x^3 - 24 \ln x$ has a minimum point.
Find its x-coordinate.

33 Show that the graph of $f(x) = \sin x - \cos x$ has a local maximum point at $\left(\frac{3\pi}{4}, \sqrt{2}\right)$.

34 An open box has a square base of side x cm and height $\frac{32}{x^2}$ cm.

Show that the surface area of the box is given by $S = x^2 + \frac{128}{x}$, and find the minimum possible surface area of the box. Show that the value you have found is a minimum.

35 Find the x-coordinate of the point of inflection on the curve with equation $y = 3x^5 - 10x^4 + 8x + 2$.

36 The displacement, s m, of an object at time t seconds, is given by $s = 3\sin(5t)$.
   Find the acceleration of the object after 2 seconds.

37 The displacement, s of an object is given by $s = 3\mathrm{e}^{-0.2t}$, where s is measured in metres and t in seconds.
   Find the speed of the object after 4 seconds.

38 The velocity of an object, measured in m s^{-1}, is given by $v = \dfrac{1}{\sqrt{t+3}}$. When $t = 2$, the displacement of the object from the origin is 4 m.
   Find the displacement from the origin when $t = 5$.

39 The velocity of an object at time t seconds is given by $v = 2\cos(0.4t)\,\mathrm{m\,s}^{-1}$.
   Find the distance travelled by the object in the first 10 seconds.

40 Find $\displaystyle\int 2x^{-\frac{2}{3}} + \frac{4}{3x}\,\mathrm{d}x$.

41 Find $\displaystyle\int \left(2\mathrm{e}^{4x} + 3\mathrm{e}^{-\frac{1}{3}x}\right)\mathrm{d}x$.
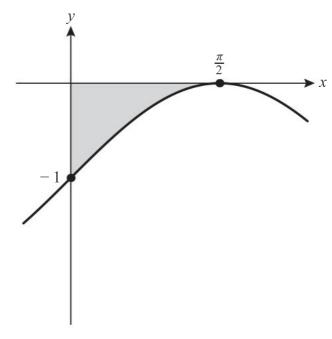
42 Find the following integrals:

a $\int 4\cos x\,\sin^2 x\,\mathrm{d}x$

b $\int \dfrac{x}{x^2+3}\,\mathrm{d}x$
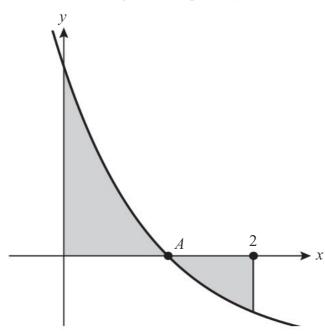
43 Evaluate $\int_0^{\frac{\pi}{6}} \sin 2x\,\mathrm{d}x$.

44 Evaluate $\int_0^2 2\mathrm{e}^{-x^3}\,\mathrm{d}x$.

45 The curve in the diagram has equation $y = \sin x - 1$.
Find the exact value of the shaded area.

46 The curve in the diagram has equation $y = 3e^{-x} - 1$.

a Find the exact coordinates of A.

b Find the shaded area.

47 a Find the coordinates of the points of intersection of the curves $y = 2e^{0.5x}$ and $y = x + 5$.

b Find the area enclosed by the two curves.

Paper plan

Syllabus Section	Syllabus Section	Description	Book Section	Mastery — Covered	Mastery — Revised	Mastery — Confident	Book P1	Book P2	A1	A2	B1	B2	C1	C2
Number and algebra Core	S1.1	Standard Form	1B						1					2a
	S1.2	Arithmetic sequences and series	2A							2,8	3		1	6
	S1.3	Geometric sequences and series	2B							8				10
	S1.4	Financial applications of geometric sequences	2C							8				
	S1.5a	Exponents with integer coefficients	1A								1			
	S1.5b	Introduction to logarithms	1C									6		
Number and algebra SL	S1.6	Deductive proof	11A						6a			3		
	S1.7a	Rational exponents	12A						1b					
	S1.7b	More logarithms	12B				4a				7	6		
	S1.8	Infinite geometric sequences	13A											
	S1.9	Binomial theorem	13B						2			5		
Functions Core	S2.1	Equations of straight lines	4A				2	8, 10b						
	S2.2	Concepts of functions	3A					5		3				
	S2.3	The graph of a function	3B					4					5	
	S2.4	Key features of graphs	3B					4					10c	9d
Functions SL	S2.5a	Composite functions	14A				5a		5					
	S2.5b	Finding inverse functions	14B				10e	5c		3c	9e			
	S2.6	Quadratic functions	15A				8a							
	S2.7a	Quadratic equations and inequalities	15B						8b		1		9a,d	
	S2.7b	Quadratic discriminant	15C				8d						9c	
	S2.8	Rational functions	16B					4a		3	9a		6	
	S2.9	Exponential and logarithmic functions	16C						5		10b			
	S2.10a	Solving equations analytically	17A				4b		7		9c		6, 9a	
	S2.10b	Solving equations graphically including using technology	17B					2c	8f			6		
	S2.11	Transformations of graphs	16A				8b		3a	3a	10b		4	4a

Practice Paper Coverage: A1, A2, B1, B2, C1, C2

Syllabus Section	Description	Book Section	Mastery — Covered	Mastery — Revised	Mastery — Confident	Book P1	Book P2	A1	A2	B1	B2	C1	C2
Geometry and trigonometry Core													
S3.1a	Distances and midpoints	4B				2c	8d, 10c					9b,e	
S3.1b	Volume and surface area of 3D solids	5A					7		1a				2, 9b,c
S3.1c	Angle between intersecting lines and planes	5B							1b				9b,e
S3.2a	Trigonometry in right-angled triangles	5B					7		1b				9a
S3.2b	Trigonometry in non-right-angled triangles	5B					1, 3		4		4		3
S3.3	Applications of trigonometry	5C					7		4		4		
S3.4	Radian measure and application to circles	18A					1		4	5	1	2	
Geometry and trigonometry SL													
S3.5	Extending definitions of trig. ratios	18B				6a			6			7	
S3.6	Trigonometric identities	18C				6b		7		5		10a	5
S3.7	Graphs of trigonometric functions	18D										10a	
S3.8	Solving trigonometric equations	18E						7			4	10b	
Statistics and probability Core													
S4.1	Sampling	6A									7abcd		
S4.2	Statistical diagrams	6C				1						3	
S4.3	Summary statistics	6B							7a	4			1
S4.4	Correlation and regression	6D							7cd		7ef		6c
S4.5	Definitions in probability	7A											
S4.6	Probability techniques	7B				3, 9		9c					7,8f
S4.7	Discrete random variables	8A						9		8			
S4.8	Binomial distribution	8B					9c		9bc		8		8d,e
S4.9	Normal distribution	8C					9b						8b,c
Statistics and probability SL													
S4.10	x-on-y regression	19A							9d		7fg		
S4.11	Formal conditional probability	19B						4					7,8c
S4.12	Standardizing normal variables	19C					9a		9a				8a

Syllabus Section	Description	Book Section	Mastery			Practice Paper Coverage							
			Covered	Revised	Confident	Book P1	Book P2	A1	A2	B1	B2	C1	C2
Calculus Core — S5.1a	Concept of a limit	9A									2		
S5.1b	Interpretation of derivatives	9A											
S5.2	Increasing and decreasing functions	9B				5b, 10a		8c				10a,c	
S5.3	Derivative of polynomials	9C						8		2		5, 8b	
S5.4	Tangents and normal	9D					2b, 10a	3b		9c			
S5.5a	Integration as antidifferentiation	10A						3a				8d	
S5.5b	Definite integrals and areas using technology	10B											4b
Calculus SL — S5.6a	Derivatives of standard functions	20A					2a		6		9ab	10a	
S5.6b	Chain rule	20B					2a				9ab	10b	
S5.6c	Product and quotient rules	20C				10b				9c	9ab	10a	
S5.7	Second derivative	20D				10c,d		8d				5	
S5.8a	Stationary points and optimization	20E				5c	10d	8e	6		9a		
S5.8b	Points of inflection	20F				5d, 10c,d					9b	5	
S5.9	Kinematics	21C					6		5			8	
S5.10	Integration of standard functions	21A						6b		10c	9c		
S5.11	Further definite integrals and areas	21B				7	4b			10c	9c	7, 8d,e	4b

Practice examination papers

Mathematics: analysis and approaches
Standard level
Paper 1
Practice Set A

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula booklet is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

1 *[Maximum mark: 5]*

    a   Write $3.4 \times 10^{23} + 2.8 \times 10^{22}$ in the form $a \times 10^k$ where $1 \leqslant a < 10$ and k is an integer.   *[2]*

    b   Evaluate $8^{-\frac{2}{3}}$ in the form $a \times 10^k$ where $1 \leqslant a < 10$ and k is an integer.   *[3]*

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2 *[Maximum mark: 5]*

    a Find the binomial expansion of $(2 + x)^5$. *[3]*

    b Hence find an approximate value for 2.01^5 correct to 4 decimal places. *[2]*

3 *[Maximum mark: 7]*

The graph of y against x has a gradient at any point equal to $3x^2 + 2$. It passes through the point $(0, 1)$.

a Find the equation of the graph.   *[3]*

b Find the equation of the normal to the graph passing through $(0, 1)$ in the form $ay + bx = c$ where
     a, b and c are integers.   *[4]*

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4 *[Maximum mark: 5]*

Let A and B be events such that $P(A) = 0.4$ and $P(B) = 0.5$, $P(A \cup B) = 0.6$. Determine, with justification, if A and B are independent. *[5]*

5 *[Maximum mark: 7]*

The functions f and g are defined such that $f(x) = \ln x$ for $x > 0$ and $g(x) = e^{x+1}$.

a Find the range of g(x). *[3]*

b Solve $(g \circ f)\,(x) = (f \circ g)\,(x)$. *[4]*

6 *[Maximum mark: 7]*

   a Prove that
$$\frac{1}{x-1} - \frac{1}{x} \equiv \frac{1}{x^2 - x}$$
[2]

   b Hence find
$$\int_3^4 \frac{2}{x^2 - x}\, dx$$

     leaving your answer in the form q where q is a rational number.
[5]

7 *[Maximum mark: 6]*

Solve $\cos 2x + \sin x = 0$ for $0 \leqslant x < 2\pi$.   *[6]*

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Do **not** write solutions on this page

Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

8 *[Maximum mark: 13]*

Let $f(x) = x^3 - 6x^2 + 9x + 4$.

  a Find $f'(x)$.   *[2]*

  b Find all solutions of $f'(x) = 0$.   *[3]*

  c Find the values of x for which $f(x)$ is increasing.   *[2]*

  d Find $f''(x)$.   *[1]*

  e Hence find and classify the stationary points on the graph of $y = f(x)$.   *[3]*

  f The equation $f(x) = k$ has three solutions. Find the possible values of k.   *[2]*

9 *[Maximum mark: 12]*

A fair four-sided dice is rolled twice. S is the sum of the scores.

  a Copy and complete the probability distribution of S.   *[2]*

s	2	3	4	5	6	7	8
$P(S = s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$			

  b Find the expected value of S.   *[2]*

  c Given that the total is more than 4, find the probability that it is more than 6.   *[3]*
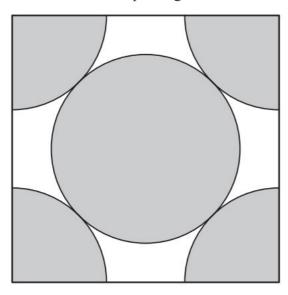
Eric plays a game where he rolls a fair four-sided dice twice. If the score is four or less, he loses and pays \$1. If he scores 5 or more, he receives \$k.

  d Find the value of k if the game is fair.   *[5]*

10 *[Maximum mark: 13]*

Metal rods are modelled as perfect cylinders with radius 1 cm. They are packed into a box in two different ways.

In method 1 the repeating unit is shown in the following diagram:
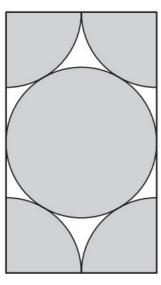
The repeating unit contains four quarter circles and one full circle.

a   Explain why the diagonal of the square has length 4 cm.   *[1]*

b   Find the proportion of the box which is filled with metal.   *[4]*

c   State one assumption required in your answer to part **b**.   *[1]*

In method 2 the repeating unit is shown in the following diagram:

d   Find the proportion of the box which contains metal in method 2.   *[5]*

e   Determine, with justification, whether method 1 or method 2 pack more rods into the same box.   *[2]*

Mathematics: analysis and approaches
Standard level
Paper 2
Practice Set A

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula booklet is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

1　*[Maximum mark: 6]*

A right pyramid has a square base with sides of length 4 cm. It has a height of 6 cm.

a　Find the volume of the pyramid.　　　　　　　　　　　　　　　　　　*[2]*

b　Find the acute angle between the sloping edge of the pyramid and the base.　*[4]*

2 *[Maximum mark: 5]*

The sum of the first n terms of an arithmetic sequence is given by $9n - 2n^2$. Find the first term and the common difference. *[5]*

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2 *[Maximum mark: 5]*

The sum of the first n terms of an arithmetic sequence is given by $9n - 2n^2$. Find the first term and the common difference.

3 *[Maximum mark: 8]*

Let $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{1}{x}$.

a Write down the transformation which maps the graph of $y = g(x)$ to $y = f(x)$. *[1]*

b Hence or otherwise write down the equations of the asymptotes of $y = f(x)$. *[2]*

c Find $f^{-1}(x)$. *[3]*

d Write down the equations of the asymptotes of $y = f^{-1}(x)$. *[2]*

4 *[Maximum mark: 6]*

A triangle ABC has sides $AB = 8$, $BC = 6$ and angle $BAC = \dfrac{\pi}{6}$ radians.

Find the two possible values of angle ABC. *[6]*

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5 *[Maximum mark: 6]*

A particle is moving in a straight line with velocity given by $v = 2e^{-t^2} - 1$ for $0 \le t \le 4$ where v is in $m\,s^{-1}$ and t is in s.

    a   Find the time at which the speed is zero.   *[2]*

    b   Find the displacement from the initial position when $t = 4$.   *[2]*

    c   Find the total distance travelled in the first four seconds.   *[2]*

6 *[Maximum mark: 6]*

A semicircle has diameter *AB* with length 2. The centre of *AB* is labelled O.

The point *P* is on the curved edge of the semicircle and *POA* is the acute angle θ.

A rectangle is drawn with one edge on part of the line segment *AB*, one corner at *P* and another corner also on the curved part of the semicircle.

a Prove that the area of the rectangle is given by $k \sin 2\theta$ where k is a constant to be determined. *[3]*

b Hence find the largest area of such a rectangle. *[3]*

Do **not** write solutions on this page

Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.
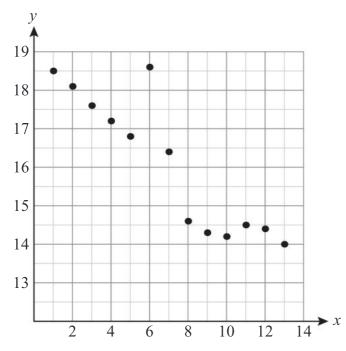
7 *[Maximum mark: 14]*

Kwami keeps a record of his best 5000 m time each week (y minutes) in the xth week after he starts training for a competition season. The results are shown below:

x	1	2	3	4	5	6	7	8	9	10	11	12	13
y	18.5	18.1	17.6	17.2	16.8	18.6	16.4	14.6	14.3	14.2	14.5	14.4	14

  a **i** Find the mean of Kwami's best times each week.

      ii Find the standard deviation in Kwami's best times each week.

      iii Find Pearson's product-moment correlation coefficient for this data. What type of correlation is suggested by this value? *[4]*

The results are illustrated in the following scatter diagram.

  b **i** Competitions occurred every week from week n until week 12. Athletes generally have improved performance in competitions. Use the graph to suggest the value of n.

      ii During one of the weeks before competitions began, Kwami was ill. Use the scatter graph to suggest which week this was. *[2]*

For the rest of this question, the result from the week where Kwami was ill should be excluded.

  c **i** Create a piecewise linear model to predict y for a given x.

      ii Compare and contrast, in context, the coefficients of x in each part of the linear model. *[5]*

  d Use your model to predict the time Kwami would have achieved in the week he was ill if he had not been ill. *[2]*

  e Explain why it would not be valid to use this model to predict Kwami's times in the following season. *[1]*

8 *[Maximum mark: 13]*

Almira is considering two different savings schemes. Both schemes involve an initial investment of $1000 in an account.

In scheme A, at the end of each year $50 is added to the account.

In scheme B, at the end of each year 4% compound interest is added to the account.

a How much will be in Almira's account at the end of the fifth year after investment in

    i scheme A

    ii scheme B.

    Give your answers correct to two decimal places. *[4]*

b What annual compound interest rate would achieve that same outcome for Almira as investing in scheme A for 5 complete years? *[2]*

c Almira wants to invest for n complete years. For what values of n would Almira be better off investing in scheme B? *[3]*

d Almira estimates that there is 2.5% depreciation each year. How long would Almira need to save in scheme B to use her savings to purchase something currently valued at $1400? *[4]*

9 *[Maximum mark: 16]*

The results in an intelligence test are normally distributed with a mean of 100. It is designed so that only 5% of people have a score of more than 150.

a Find the standard deviation of results in the test. *[4]*

To enter a high intelligence society, people need to have a score of at least 150. Five people are chosen at random to take the test.

b Find the probability that at least two of them qualify to enter the high intelligence society. *[4]*

c Find the probability that the fifth person to take the test is the second person to attain a score of at least 150. *[3]*

People with a score of more than 170 in the test are allowed to enter a merit stream within the society.

d What percentage of the society are members of the merit stream? *[4]*

e State one assumption required in your answer to part **d**. *[1]*

Mathematics: analysis and approaches
Standard level
Paper 1
Practice Set B

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
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Section A

1 *[Maximum mark: 5]*

    a Evaluate $\dfrac{8^{-3} \times 4^{6}}{2^{2}}$.   *[3]*

    b Simplify $\dfrac{x^{4}y^{2} + x^{3}y}{x^{2}y}$.   *[2]*

2 *[Maximum mark: 4]*

$p(x) = x^3 + 3x^2 - x + 4$

Find the gradient of the tangent to $p(x)$ at the point where $x = -2$. *[4]*

3 *[Maximum mark: 5]*

An arithmetic sequence has first term a and common difference d.

The sixth term of the sequence is 10. The sum of the first eight terms of the sequence is 62.

Find a and d.   *[5]*

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4 *[Maximum mark: 5]*

Solve the inequality $2x^2 + 5x - 12 > 0$. *[5]*

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5 *[Maximum mark: 5]*

$\sin \theta = \dfrac{1}{3}$, where $\dfrac{\pi}{2} < \theta < \pi$.

a   Find the exact value of $\cos \theta$.   *[3]*

b   Hence find the exact value of $\tan \theta$.   *[2]*

6 *[Maximum mark: 7]*

The scores out of 10 in a mathematics test are summarized in the table:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	1	0	1	1	2	2	3	2	1	1

a Find the median score. *[1]*

b Find the interquartile range. *[3]*

c Determine, with clear calculations, whether any of the scores are outliers. *[3]*

7 *[Maximum mark: 6]*

Solve $5^{x+2} = 3^{x-1}$, giving your answer in the form $\dfrac{\ln a}{\ln b}$, where a and b are rational numbers to be found. *[6]*

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Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

8 *[Maximum mark: 14]*

The table below is the probability distribution for the score obtained in a game of chance:

x	0	1	2	3	4
$P(X=x)$	0.1	a	b	0.2	0.15

A player is charged \$2 to play and they win, in dollars, the score they achieve in the game.

a Show that $a + b = 0.55$. *[2]*

b Given that the game is fair, find a and b. *[3]*

Dorian plays the game once.

c Given that his score is odd, find the probability that he makes a profit. *[3]*

Zhuo plays the game twice.

d Find the probability he gets a higher score in the second game than the first. *[3]*

e Find the probability he makes a loss overall. *[3]*

9 *[Maximum mark: 16]*

$f(x) = \dfrac{2x - 5}{x - 1}, x \neq 1.$

a Sketch the graph of $y = f(x)$ labelling all axis intercepts and asymptotes. *[4]*

b State the range of f. *[1]*

The normal to $y = f(x)$ at the point where $x = 2$ intersects the curve again at the point P.

c Find the x-coordinate of P. *[6]*

d Find the inverse function f^{-1}. *[3]*

e State the

   i domain of f^{-1}

   ii range of f^{-1}. *[2]*

10 *[Maximum mark: 13]*

Let C_1 be the curve $y = e^x$ and C_2 the curve $y = 6 - 5e^{-x}$.

a Find the x-coordinate of the points of intersection of C_1 and C_2. *[4]*

b On the same set of axes, sketch the graphs of C_1 and C_2. *[4]*

c Find the area enclosed between C_1 and C_2, giving your answer in the form $a + \ln b$, where a and b are integers to be found. *[5]*

Mathematics: analysis and approaches
Standard level
Paper 2
Practice Set B

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
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Section A

1 *[Maximum mark: 4]*

The sector *OAB* has area $15\,\text{cm}^2$.

The perimeter of the sector is four times the length of the arc *AB*.

Find the radius, *r*. *[4]*

2 *[Maximum mark: 6]*

    a   Find an expression for the gradient of the chord joining the point $(1, 0)$ to the point $(x, \ln x^2)$.   *[2]*

    b   Use technology to find the limit of the gradient of this chord as x tends towards 1.   *[2]*

    c   Explain the significance of your answer to part **b**.   *[2]*

3 *[Maximum mark: 5]*

Prove that if $n > 2$ is an even integer, then $2^n - 1$ is not prime. *[5]*

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4 *[Maximum mark: 8]*

The London Eye is an observation wheel with a diameter of 120 m that rotates once every 30 minutes.

The pods that carry customers are arranged around the rim of the wheel.

A particular pod starts at the lowest point of the circle, 2 m above ground level. The height, *h* metres, of that pod at time *t* minutes can be modelled by the function $h = a\cos(bt) + c$.

a Find the values of *a*, *b* and *c*.   *[5]*

b Find the length of time for which the pod is higher than 50 m above ground level in any one revolution of the wheel.   *[3]*

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5 *[Maximum mark: 7]*

The sum of the first three terms of a geometric sequence is 74, and the sum to infinity is 128.

a Find the value of the common ratio, *r*, of the sequence. *[4]*

b Find the sum of the first six terms of the sequence. *[3]*

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6 *[Maximum mark: 6]*

Find the set of values of a for which there are two distinct real solutions to the equation $\log_2 x + 3 = \log_x a$.   [6]

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Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

7 *[Maximum mark: 14]*

Fola is researching the possible relationship between the height and weight of staff at his school.

He initially decides to select his sample by taking an alphabetic list of all staff and selecting every 10th person on the list.

a i Name this sampling technique.

  ii Explain why this does not produce a simple random sample. *[2]*

Fola then decides to change his sampling technique by taking a stratified sample of men and women.

He wants a sample size of 12 and knows there are 46 men and 63 women at the school.

b i Find the number of men he should include in his sample.

  ii State the sampling method Fola needs to employ to select the particular men and women.

  iii State how quota sampling differs from stratified sampling. *[4]*

Fola collects the following data:

Height (cm)	153	158	161	162	164	165	167	172	175	179	184	190
Weight (kg)	52.4	54.6	59.7	57.1	58.5	74.2	62.8	73.1	82.3	60.2	74.3	86.6

c Find Pearson's product moment correlation coefficient and interpret this value in context. *[2]*

d Use an appropriate regression line to estimate

  i the height of a person with weight $68\,\text{kg}$

  ii the weight of a person with height $170\,\text{cm}$. *[4]*

e Suggest two ways Fola could improve the reliability of any predictions made from linear regression for this population. *[2]*

8 *[Maximum mark: 16]*

A telesales worker has constant probability of 0.04 of a call resulting in a sale.

a Find the probability of achieving exactly two sales in the first 10 calls made. *[2]*

b Find the probability of achieving at least two sales in the first 10 calls made. *[2]*

c i Find the number of calls they need to make in a day to average two sales per day.

  ii In this case, find the variance of the number of sales achieved. *[4]*

d In a five-day week, find the probability they achieve at least two sales per day on more than one day. *[4]*

e Find the least number of calls they need to make in order that the probability of making at least one sale is greater than 95%. *[4]*

9 *[Maximum mark: 14]*

Part of the graph of $y = \sin x \, e^{\cos x}$ is shown below.

a Show that the x-coordinates of the turning points satisfy $\sin x \tan x = 1$.   *[4]*

b Show that the x-coordinates of any points of inflection satisfy $\sin 2x = 0$.   *[5]*

c Show that the area of the shaded region area enclosed below the curve and above the x-axis is given by $a e^{-1} + b e$, where a and b are constants to be found.   *[5]*

**Mathematics: analysis and approaches
Standard level
Paper 1
Practice Set C**

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula booklet is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

1 *[Maximum mark: 4]*

An arithmetic sequence has first term 7 and the sum of the first 20 terms is 640. Find

   a the 20th term of the sequence   *[2]*

   b the 39th term of the sequence.   *[2]*

2 *[Maximum mark: 5]*

A sector of a circle has radius 10 cm. The angle at the centre of the sector is θ radians. The area of the sector is 75 cm².

a Find the value of θ. *[2]*

b Find the perimeter of the sector. *[3]*

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3 *[Maximum mark: 5]*

Roshni collected 200 apples from her orchard. The cumulative frequency graph below shows their mass in grams.
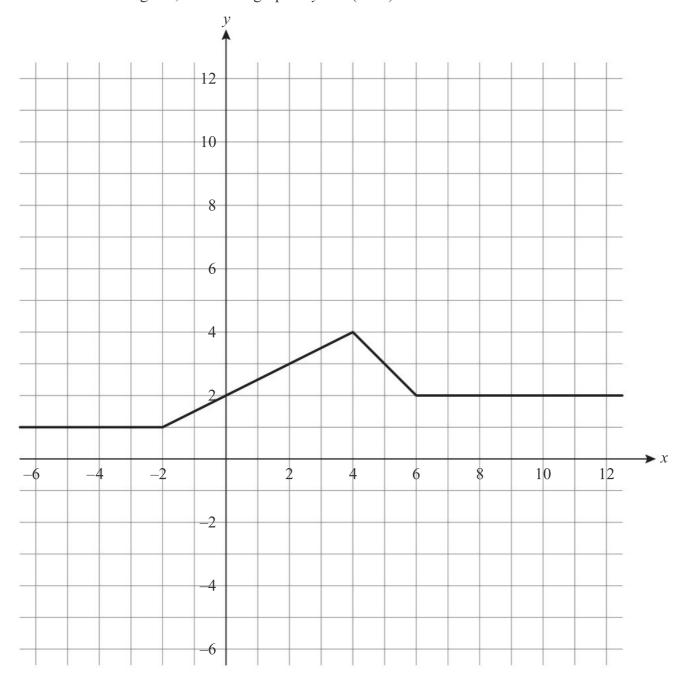
a Estimate how many apples weigh more than 90 g.   *[2]*

b The heaviest 15% of the apples are going to be sent to the local restaurant. Estimate the lowest weight of an apple sent to the restaurant.   *[3]*

4 *[Maximum mark: 6]*

a The diagram shows the graph of $y = f(x)$.

On the same diagram, sketch the graph of $y = 2f(x - 3)$. *[2]*

b The graph of $y = 2 \times 3^x$ is stretched horizontally with scale factor $\frac{1}{2}$ and then stretched vertically with scale factor 6.

Find the equation of the resulting graph in the form $y = A \times B^x$. *[4]*

5 *[Maximum mark: 7]*

The graph of $y = 2x^3 - ax^2 + x + 2b$ has a point of inflection at $(2, -6)$. Find the values of a and b.   *[7]*

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6 *[Maximum mark: 6]*

Find the exact solution of the equation $e^{2x} - 2e^x - 8 = 0$. *[6]*

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7 *[Maximum mark: 6]*
Find the exact value of $\int_0^{\sqrt{\frac{\pi}{3}}} 8x \cos(x^2)\, \mathrm{d}x$. *[6]*
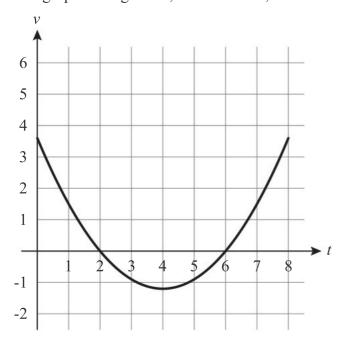
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Do **not** write solutions on this page

Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

8 *[Maximum mark: 14]*

The velocity of a particle, $v\,\mathrm{m\,s^{-1}}$, at time t seconds is given by $v = 0.3t^2 - 2.4t + 3.6$.

The graph of v against t, for $0 \leqslant t \leqslant 8$, is shown below.
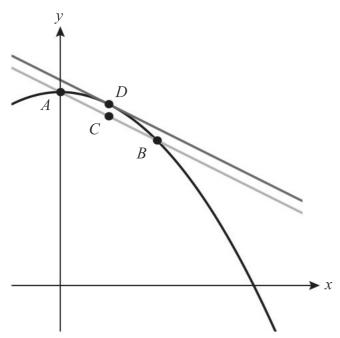
a Find the speed of the particle after 3 seconds. *[2]*

b Find the time when the acceleration is $2.4\,\mathrm{m\,s^{-2}}$. *[3]*

c State two times when the particle changes direction. *[1]*

d Find the distance travelled by the particle in the first

   i 2 seconds *[4]*

   ii 3 seconds. *[2]*

e What is the displacement of the particle from the starting position when $t = 3$ seconds? *[2]*

9 *[Maximum mark: 13]*

The diagram shows the curve with equation $y = 4 - x^2$. The line $y = 4 - x$ intersects the curve at the points A and B. The point C is the midpoint of AB. The line $y = k - x$ is tangent to the curve at point D.

a	Find the coordinates of A and B.	*[3]*
b	Find the coordinates of C.	*[1]*
c	Find the value of k.	*[5]*
d	Find the x-coordinate of D.	*[2]*
e	Find the distance CD.	*[2]*
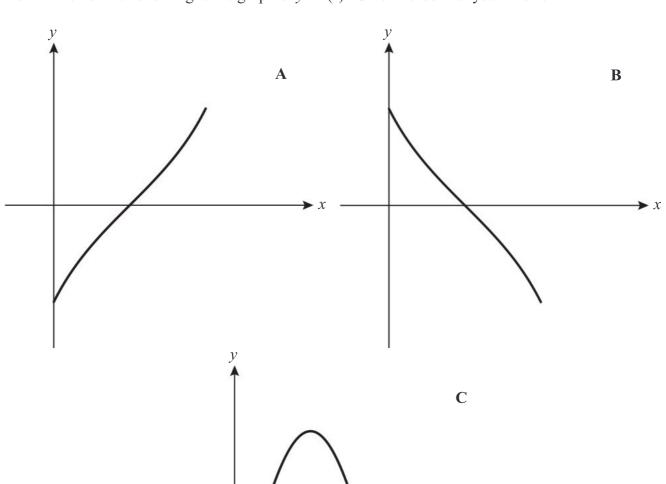
10 *[Maximum mark: 14]*

A function is defined on the domain $0 \leqslant x \leqslant \frac{\pi}{2}$ by

$$f(x) = \frac{\sin x - \cos x}{\sin x + \cos x}.$$

a i Show that $f'(x) = \dfrac{2}{1 + \sin 2x}$. [6]

    ii State the minimum and maximum values of $1 + \sin 2x$. Hence explain why the function is increasing for all values of x in its domain. [2]

b Find the coordinates of the point of inflection on the graph of $y = f(x)$. [4]

c Which of the following is the graph of $y = f(x)$? Give a reason for your answer. [2]

A

B

C

Mathematics: analysis and approaches
Standard level
Paper 2
Practice Set C

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
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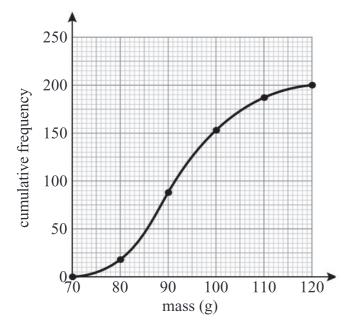- The maximum mark for this examination paper is **[80 marks]**.

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1   *[Maximum mark: 6]*

The heights of 30 flowers, measured in cm, are summarized in the table.

height (cm)	8–12	12–15	15–20	20–25	25–28
frequency	5	6	8	7	4
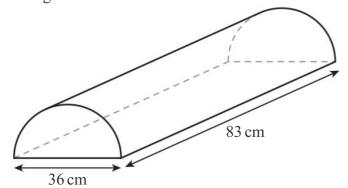
  a   Estimate the mean and the standard deviation of the 30 flowers.   *[3]*

The measurements are converted into inches, where 1 inch = 2.54 cm.

  b   Find the mean and variance of the heights in inches.   *[3]*

2 *[Maximum mark: 6]*

A metal bar is in the shape of a prism with a semicircular cross-section. The dimensions are shown in the diagram.

83 cm

36 cm

a Find the volume of the bar. Give your answer in cm^3, in the form $a \times 10^k$ where $1 \leq a < 10$
and $k \in \mathbb{Z}$. *[3]*
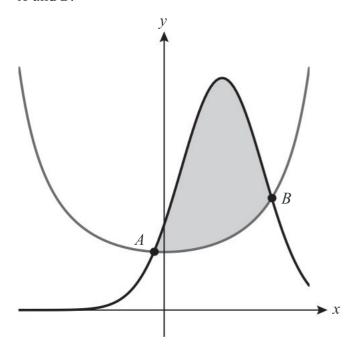
The bar is melted down and all the metal used to make a sphere.

b Find the radius of the sphere. *[3]*

3 *[Maximum mark: 5]*

A triangle has sides $AB = 6.8$ cm, $BC = 4.7$ cm and $CA = 9.1$ cm.

a Find, in degrees, the size of the angle $A\hat{B}C$. *[3]*

b Find the area of the triangle. *[2]*

4   *[Maximum mark: 6]*

The curves in the diagram have equations $y = 4e^{-(x-1)^2}$ and $y = \dfrac{x}{\sin x}$. The curves intersect at the points A and B.

a   Find the coordinates of A and the coordinates of B.   *[3]*
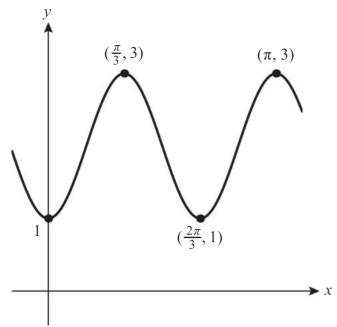
b   Find the shaded area.   *[3]*

5 *[Maximum mark: 5]*

The graph in the diagram has equation $y = d + \sin(b(x - c))$, where x is in radians and $0 < c < \dfrac{\pi}{2}$.

Find the exact values of b, c and d.   *[5]*

6 *[Maximum mark: 6]*

Rin has started a new martial arts club. The number of members during the first six months is shown in the table:

Month	1	2	3	4	5	6
Number of members	26	34	44	51	59	66

Rin thinks that the number of members can be modelled by a sequence which is approximately arithmetic.

a Find an average increase in the number of members over the first six months. *[2]*

b Use this model, with the first term equal to 26, to predict the number of members at the end of the year (in month 12). *[2]*

Gabor thinks that a better way to predict the number of members in future is to use a regression line.

c Use the data from the table to find the equation of the regression line and use it to predict the number of members at the end of the year. *[2]*

7 *[Maximum mark: 6]*

Events A and B are such that:

$P(B) = \frac{1}{6}$, $P(A \cup B) = \frac{1}{5}$ and $P(A|B) = 4P(A)$.

a Explain why A and B are not independent. *[1]*

b Find $P(A \cap B)$. *[5]*

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Do **not** write solutions on this page

Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

8 *[Maximum mark: 14]*
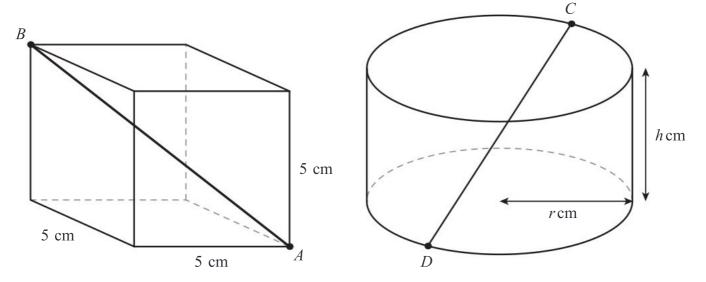
The times taken by children to complete a race can be modelled by a normal distribution with mean μ minutes and standard deviation 2.5 minutes. It is known that 12% of children take more than 8.5 minutes to complete the race.

a Find the value of μ. *[3]*

b Find the probability that a randomly selected child completes the race in less than 9.2 minutes. *[1]*

c Given that a randomly selected child completes the race in less than 9.2 minutes, find the probability that they complete the race is less than 8.3 minutes. *[2]*

20 randomly selected children run the race.

d Find the expected number of children who complete the race in less than 9.2 minutes. *[2]*

e Find the probability that at least 18 of the 20 children complete the race in less than 9.2 minutes. *[3]*

Two separate groups of 20 children run the race.

f Find the probability that in exactly one of the groups, at least 18 children complete the race in less than 9.2 minutes. *[3]*

9 *[Maximum mark: 14]*

The diagram shows a cube with side 5 cm and a cylinder with base radius r cm and height h cm.

a Find the length of AB. *[2]*

b Find the angle that the line AB makes with the horizontal base of the cube. *[2]*

The cylinder and the cube have the same volume.

c Show that the surface area of the cylinder is given by $\frac{250}{r} + 2\pi r^2$. *[4]*

d Compare the minimum possible surface area of the cylinder to the surface area of the cube. *[3]*

e Assume the cylinder has the minimum possible surface area found in part **d**. The line CD is the longest line that can be drawn between the bottom base and the top base of the cylinder. Find the angle that this line makes with the base of the cylinder. *[3]*

10 *[Maximum mark: 12]*

Denote by u_n the nth term of a geometric sequence with first term 1 and common ratio $\frac{1}{2}$.

a Write down the first four terms of the sequence u_n. *[1]*

b Find the sum to infinity of the series $u_1 + u_2 + u_3 + \cdots$ *[2]*

The sequence v_n is defined by:

$$v_1 = 1 \text{ and } v_{n+1} = \frac{1}{3}v_n + 1.$$

(So $v_2 = \frac{1}{3}v_1 + 1, v_3 = \frac{1}{3}v_2 + 1$ etc.)

c Find the first four terms of the sequence v_n. *[2]*

d Suggest the value for the limit of the sequence v_n. *[2]*

The sequence w_n is defined by $w_n = \sum_{k=1}^{n} 2 \times \left(\frac{1}{3}\right)^k$.

e Write w_n in the form $w_n = A - B^n$, where A and B are constants to be found. *[3]*

f Find the limit of the sequence w_n. *[2]*

Practice Set A: Paper 1 Mark scheme

SECTION A

1 **a**   Write both numbers using the same power of 10   (M1)
3.68×10^{23} A1

[2 marks]

    b $8^{-\frac{2}{3}} = \dfrac{1}{\left(\sqrt[3]{8}\right)^2}$ (M1)

        $= \dfrac{1}{4}$ (A1)

        $= 2.5 \times 10^{-1}$ A1

[3 marks]
Total [5 marks]

2 **a**   Attempt to find binomial coefficients   (M1)
Powers of 2 and x that sum to 5   (M1)
$32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$ A1

[3 marks]

    b $x = 0.01$ M1
32.8080   A1

[2 marks]
Total [5 marks]

3 **a** $y = \int 3x^2 + 2 \, dx$ (M1)
     $= x^3 + 2x + c$ (A1)
When $x = 0$, $y = 1$ so $y = x^3 + 2x + 1$ A1

[3 marks]

    b   When $x = 0$, gradient of tangent is 2.   (M1)

       So gradient of normal is $-\dfrac{1}{2}$ (A1)

       $y = -\dfrac{1}{2}x + 1$ (M1)
       $2y + x = 2$ A1

[4 marks]
Total [7 marks]

4 $P(A \cap B) = P(A) + P(B) - P(A \cup B)$ (M1)
$= 0.4 + 0.5 - 0.6 = 0.3$ A1
$P(A)P(B) = 0.4 \times 0.5 = 0.2$ M1A1

$P(A)P(B) \neq P(A \cap B)$ so not independent   R1

Total [5 marks]

5 **a**   Sketch of $y = g(x)$ (M1)
$g(x) > 0$ A1A1

Note: A1 for strict inequality. A1 for asymptote at 0.   *[3 marks]*

    b $(g \circ f)(x) = e^{\ln x + 1}$ M1
      $= e^{\ln x} \, e^1$ A1
      $= ex$
      $(f \circ g)(x) = \ln(e^{x+1})$
      $= x + 1$ A1
      $ex = x + 1$
      $x(e - 1) = 1$
      $x = \dfrac{1}{e-1}$ A1

[4 marks]
Total [7 marks]

6 a LHS $= \dfrac{x}{x(x-1)} - \dfrac{x-1}{x(x-1)}$ M1

$= \dfrac{x-x+1}{x(x-1)}$ A1

$= \dfrac{1}{x^2-x}$ AG

[2 marks]

b $\displaystyle\int_3^4 \dfrac{2}{x^2-x}\,dx = \int_3^4 \dfrac{2}{x-1} - \dfrac{2}{x}\,dx$ (M1)

$= \left[2\ln|x-1| - 2\ln|x|\right]_3^4$ (A1)

Note: Condone absence of modulus signs

$= (2\ln 3 - 2\ln 4) - (2\ln 2 - 2\ln 3)$ (M1)

$= 4\ln 3 - 2\ln 4 - 2\ln 2$

$= \ln 81 - \ln 16 - \ln 4$

$= \ln 81 - 2\ln 4 - \ln 4$ (M1)

$= \ln 81 - 3\ln 4$

$= \ln 81 - \ln 64$

$= \ln\left(\dfrac{81}{64}\right)$ A1

[5 marks]

Total [7 marks]

7 $\cos 2x = 1 - 2\sin^2 x$ M1

$0 = 2\sin^2 x - \sin x - 1$

$= (2\sin x + 1)(\sin x - 1)$ M1

Note: Allow factorizing, quadratic formula or any other valid method.

$\sin x = -\dfrac{1}{2}$ or $\sin x = 1$ A1

$x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}, \dfrac{\pi}{2}$ A1A1A1

Total [6 marks]

SECTION B

8 a $3x^2 - 12x + 9$ (M1)A1

Note: Award M1 for at least one correct term

[2 marks]

b $3(x^2 - 4x + 3) = 3(x-3)(x-1) = 0$ (M1)

Note: Also award M1 for attempt at quadratic formula

$x = 3$ or $x = 1$ A1A1

[3 marks]

c $f'(x) > 0$ (M1)

$x > 3$ or $x < 1$ A1

[2 marks]

d $f''(x) = 6x - 12$ *[1 mark]*

e When $x = 1$, $y = 8$ A1

$f''(1) = -6$ so maximum   R1

When $x = 3$, $y = 4$, $f''(3) = 6$ so minimum   A1

[3 marks]

f Considering the equation graphically   (M1)

$4 < k < 8$ A1

[2 marks]

Total [13 marks]

9 a Using a lattice diagram or other systematic list   M1

s	2	3	4	5	6	7	8
$P(S = s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

A1
[2 marks]

b $E\left(S\right) = 2 \times \dfrac{1}{16} + 3 \times \dfrac{2}{16} \ldots$ (M1)

$\qquad = 5$ A1
[2 marks]

c $P(X > 4) = \dfrac{10}{16}$; $P(X > 6) = \dfrac{3}{16}$ A1

$P(X > 6 \,|\, X > 4) = \dfrac{3}{16} \div \dfrac{10}{16} = \dfrac{3}{10}$ M1A1
[3 marks]

d   If W is the winnings then:

w	-1	k
$P(W = w)$	$\frac{6}{16}$	$\frac{10}{16}$

M1A1

$\qquad E\left(W\right) = -\dfrac{6}{16} + \dfrac{10k}{16}$ A1

For the game to be fair, $E\left(W\right) = 0$ so   M1

$\dfrac{10k}{16} = \dfrac{6}{16}$

$k = 0.6$ A1
[5 marks]
Total [12 marks]

10 a   1 diameter of 2 cm and 2 radii each of 1 cm.   (R1)
[1 mark]

b   Total area of metal in each repeating unit $= 2 \times \pi \times 1^2 = 2\pi$ M1A1

If side of the square is x then $x^2 + x^2 = 16$ M1

So proportion of box filled is $\dfrac{2\pi}{8} = \dfrac{\pi}{4}$ A1
[4 marks]

c   For example, that the extra space at the edge of the box is negligible.   R1
Note: Accept any reasonable criticism of the model.
[1 mark]

d   Diagonal is 4 and width is 2.   A1

So height is $\sqrt{4^2 - 2^2} = \sqrt{12}$ M1A1

Ratio is $\dfrac{2 \times \pi \times 1^2}{2\sqrt{12}} = \dfrac{\pi}{\sqrt{12}}$ M1A1
[5 marks]

e $\sqrt{12} < \sqrt{16} = 4$ R1
Therefore method 2 can pack more rods.   A1
Note: Do not award R0A1
[2 marks]
Total [13 marks]

Practice Set A: Paper 2 Mark scheme

SECTION A

1 **a** $V = \dfrac{1}{3} \times 6 \times 4^2$ (M1)

   $32 \left(cm^3 \right)$ A1

   [2 marks]

   b Diagonal of square $= \sqrt{4^2 + 4^2} \left(= 5.66 \right)$ (M1)
   Length from corner to centre of square $= 2.83$ (A1)

   Angle is $\tan^{-1} \left(\dfrac{6}{2.83} \right)$ (M1)

   $= 64.8°$ (1.13 radians) A1

   [4 marks]
   Total [6 marks]

2 **a** EITHER
   $S_1 = 7$
   $S_2 = 10$ (A1)
   $u_1 = 7$ (A1)
   Note: Must be made clear that this is the first term. (A1)
   $u_2 = 3$ (M1)
   $d = -4$ A1

   [5 marks]

   OR

   $S_n = \dfrac{n}{2} \left(2a + (n-1)d \right) = \dfrac{d}{2} n^2 + \left(a - \dfrac{d}{2} \right) n$

   (M1)(A1)

   Comparing coefficients:

   $\dfrac{d}{2} = -2$ and $a - \dfrac{d}{2} = 9$ (M1)

   $d = -4$ A1
   $a = 7$ A1

   [5 marks]
   Total [5 marks]

3 **a** Translation 2 left A1

   [1 mark]

   b $y = 0$ A1
   $x = -2$ A1

   [2 marks]

   c $x = \dfrac{1}{y+2}$ (M1)

   $x(y + 2) = 1$ (M1)
   Note: The first method mark for switching x and y can be awarded
   before or after the second method mark.

   $y = \dfrac{1}{x} - 2 = f^{-1}(x)$ A1

   [3 marks]

   d $y = -2$ A1
   $x = 0$ A1

   [2 marks]
   Total [8 marks]

4 $\dfrac{6}{\sin(\frac{\pi}{6})} = \dfrac{8}{\sin(ACB)}$ (M1)

$\sin(ACB) = \dfrac{2}{3}$ A1

$ACB = 0.730 \text{ or } 2.41 \ (41.8° \text{ or } 138°)$ A1A1

$ABC = \pi - ACB - BAC$ (M1)

$\quad\quad = 1.89 \text{ or } 0.208 \ (108° \text{ or } 11.9°)$ A1

Total [6 marks]

5   **a**   Attempt to solve $2e^{-t^2} - 1 = 0$ graphically or otherwise   (M1)

$\quad\quad\quad$ 0.833   A1

[2 marks]

$\quad\quad$ **b** $\displaystyle\int_0^4 2e^{-t^2} - 1\, dt$ M1

$\quad\quad\quad = -2.23$ A1

[2 marks]

$\quad\quad$ **c** $\displaystyle\int_0^4 \left|2e^{-t^2} - 1\right|\, dt$ M1

$\quad\quad\quad = 3.26$ A1

[2 marks]

Total [6 marks]

6   **a**   Coordinates of P are $(\cos\theta,\ \sin\theta)$ A1

$\quad\quad\quad$ So area of rectangle is $(2\cos\theta) \times \sin\theta$ A1

$\quad\quad\quad$ Using sine double angle formula so $k = 1$.   A1

[3 marks]

$\quad\quad$ **b**   Maximizing either analytically or graphically   M1

$\quad\quad\quad \theta_{\max} = \dfrac{\pi}{4}$ (A1)

$\quad\quad\quad \text{Area}_{\max} = 1$ A1

[3 marks]

Total [6 marks]

SECTION B

7   **a**   **i**   16.1   A1

$\quad\quad\quad$ **ii**   1.73   A1

$\quad\quad\quad$ **iii**   −0.901   A1

$\quad\quad\quad\quad$ Strong negative correlation   A1

[4 marks]

$\quad\quad$ **b**   **i**   8   A1

$\quad\quad\quad$ **ii**   6   A1

[2 marks]

$\quad\quad$ **c**   **i** $y = \begin{cases} -0.361x + 18.8 & x < 8 \\ -0.0686x + 15.1 & x \geqslant 8 \end{cases}$ M1A1A1

$\quad\quad\quad$ **ii**   Both are negative so there is a trend of improving in both parts of

$\quad\quad\quad\quad$ the season.   A1

$\quad\quad\quad\quad$ The modulus of the first coefficient is larger, so there is greater

$\quad\quad\quad\quad$ improvement each week in the pre-competition training.   A1

[5 marks]

$\quad\quad$ **d** $-0.361 \times 6 + 18.8 \approx 16.6$ (minutes)   M1A1

[2 marks]

$\quad\quad$ **e**   This is an example of extrapolation which is not generally valid.   A1

[1 mark]

Total [14 marks]

8 a i $1000 + 5 \times 50 = \$1250.00$ (M1)A1

   ii $1000 \times 1.04^5 = \$1216.65$ (M1)A1

    Note: May be done using TVM so no working shown.

[4 marks]

b 4.56% (M1)A1

Note: Award M1 for any evidence of using TVM package, for example, stating principal value of 1000 and final value of 1250.

[2 marks]

c Solving $1000 + 50n = 1000 \times 1.04^n$ (M1)

Evidence of graphical, tabular or trial and error approach (M1)

$n \geqslant 12$ A1

Note: Do not accept non integer values.

[3 marks]

d Effective interest rate = 1.5% (A1)

Evidence of TVM or $1400 = 1000 \times 1.015^n$ (M1)

22.559 years (A1)

So needs 23 years A1

[4 marks]

Total [13 marks]

9 a $\Phi^{-1}(0.95) = 1.645$ from GDC (M1)(A1)

$$\frac{150 - 100}{\sigma} = 1.645$$ (M1)

$$\sigma = \frac{50}{1.645} = 30.4$$ A1

[4 marks]

b If $X = $ "number of people with score $\geqslant$ 150 out of 5"

$X \sim B(5, 0.05)$ (M1)(A1)

$$P(X \geqslant 2) = 1 - P(X \leqslant 1)$$ (M1)

$= 0.0226$ A1

[4 marks]

c We need one success in the first four, then a success (M1)

If $Y = $ 'number of people with score $\geqslant$ 150 out of 4'

$Y \sim B(4, 0.05)$

Required probability is $P(Y = 1) \times 0.05$ (M1)

$= 0.008\,57$ A1

[3 marks]

d If A is the score of a member then we require:

$P(A > 170 \mid \; > 150)$ (M1)

$$= \frac{P(A > 170 \cap A > 150)}{P(A > 150)}$$

$$= \frac{P(A > 170)}{P(A > 150)}$$ (M1)

$$= \frac{0.0107}{0.05}$$ (M1)

Note: Award M1 for evidence of using GDC to calculate any probability from a N(100, 'their value') distribution, even outside of context of conditional probability

$= 0.214$

$= 21.4\%.$ A1

[4 marks]

e That the membership of the high intelligence society is representative of the whole population. R1

[1 mark]

Total [16 marks]

Practice Set B: Paper 1 Mark scheme

SECTION A

1 **a** Write all as powers of 2   (M1)

$$\frac{2^{-9} \times 2^{12}}{2^2} = \frac{2^3}{2^2}$$
              A1

$$= 2$$
              A1

[3 marks]

    b $\dfrac{x^3 y (xy + 1)}{x^2 y}$ (M1)

$$= x(xy + 1) \text{ or } x^2 y + x$$
          A1

[2 marks]

Total [5 marks]

2 $p'(x) = 3x^2 + 6x - 1$ (M1)A1

Note: Award M1 for at least one correct term

$p'(-2) = 3(-2)^2 + 6(-2) - 1$ (M1)

      $= -1$ A1

Total [4 marks]

3 $a + (6 - 1)d = 10$

$a + 5d = 10$ A1

$\dfrac{8}{2}(2a + (8 - 1)d) = 62$
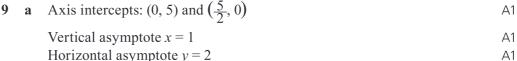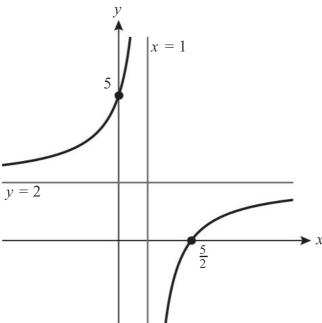
$4a + 14d = 31$ M1A1

Solve simultaneously to give:

$a = \dfrac{5}{2}, d = \dfrac{3}{2}$ A1A1

Total [5 marks]

4 $(2x - 3)(x + 4) > 0$ (M1)

Critical values $x = \dfrac{3}{2}, x = -4$ A1

Use of graph or consideration of signs of brackets   (M1)

$x < -4 \text{ or } x > \dfrac{3}{2}$ A1A1

Total [5 marks]

5 **a** $\cos^2 \theta = 1 - \left(\dfrac{1}{3}\right)^2 = \dfrac{8}{9}$ (M1)

$\cos \theta = \pm \dfrac{2\sqrt{2}}{3}$ A1

Since $\dfrac{\pi}{2} < \theta < \pi$, $\cos \theta < 0$, so $\cos \theta = -\dfrac{2\sqrt{2}}{3}$ R1

[3 marks]

    b $\tan \theta = \dfrac{\dfrac{1}{3}}{-\dfrac{2\sqrt{2}}{3}}$ (M1)

$= -\dfrac{1}{2\sqrt{2}} \text{ or } -\dfrac{\sqrt{2}}{4}$ A1

[2 marks]

Total [5 marks]

6 a 14 values so median is at 7.5th value.
Median = 6.5   A1

[1 mark]

b Q_1 is at 4th value so $Q_1 = 5$ A1
Q_3 is at 11th value so $Q_3 = 8$ (A1)
$IQR = 8 - 5 = 3$ (A1)

[3 marks]

c x an outlier if $x < Q_1 - 1.5(Q_3 - Q_1)$, so if $x < 5 - 1.5 \times 3 = 0.5$ A1

or if $x > Q_3 + 1.5(Q_3 - Q_1)$, so if $x > 8 + 1.5 \times 3 = 12.5$ A1

No data values smaller than 0.5 or larger than 12.5 so no outliers.   R1

[3 marks]
Total [7 marks]

7 $(x + 2) \ln 5 = (x - 1) \ln 3$ (M1)
$2 \ln 5 + \ln 3 = x \ln 3 - x \ln 5$ (M1)

$$\ln 75 = x \ln \frac{3}{5}$$ (M1)A1

$$x = \frac{\ln 75}{\ln \dfrac{3}{5}}$$

that is, $a = 75$, $b = \dfrac{3}{5}$ A1A1

Total [6 marks]

SECTION B

8 a $0.1 + a + b + 0.2 + 0.15 = 1$ M1A1
$a + b = 0.55$ AG

[2 marks]

b Game fair so $E(X) = 2$

$(0 \times 0.1) + a + 2b + (3 \times 0.2) + (4 \times 0.15) = 2$ (M1)
$a + 2b = 0.8$ A1
Solving simultaneously with $a + b = 0.55$
$a = 0.3$, $b = 0.25$ A1

[3 marks]

c $P(X > 2 | X \text{ odd}) = \dfrac{P(X > 2 \cap X \text{ odd})}{P(X \text{ odd})}$

$= \dfrac{P(X = 3)}{P(X = 1 \cup X = 3)}$ (M1) (M1)

$= \dfrac{0.2}{0.3 + 0.2} = 0.4$ A1

[3 marks]

d Attempt to list relevant outcomes   (M1)
(0, 1), (0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)
(2, 3), (2, 4), (3, 4)
$P(X_2 > X_1) = 0.1(0.3 + 0.25 + 0.2 + 0.15) + 0.3(0.25 + 0.2 + 0.15)$
$+ 0.25(0.2 + 0.15) + 0.2(0.15)$ (M1)
$= 0.3875$ A1

[3 marks]

e Will make loss if $X_1 + X_2 < 4$
(0, 0), (1, 1)   (M1)
(0, 1), (0, 2), (0, 3), (1, 2) AND REVERSES
$P(X_1 + X_2 < 4) = 0.1^2 + 0.3^2$

$+ 2(0.1 \times 0.3 + 0.1 \times 0.25 + 0.1 \times 0.2 + 0.3 \times 0.25)$ M1
$= 0.4$ A1

[3 marks]
Total [14 marks]

9 **a** Axis intercepts: $(0, 5)$ and $\left(\frac{5}{2}, 0\right)$ A1

Vertical asymptote $x = 1$ A1
Horizontal asymptote $y = 2$ A1
Correct shape   A1

[4 marks]

b $f(x) \neq 2$ A1

[1 mark]

c $f'(x) = \dfrac{2(x-1)-(2x-5)1}{(x-1)^2} = \dfrac{3}{(x-1)^2}$ M1A1

Note: Award M1 for attempt to use quotient or product rule
$f'(2) = 3$
Gradient of normal $= -\dfrac{1}{3}$ (A1)
$f(2) = -1$
Equation of normal: $y + 1 = -\dfrac{1}{3}(x-2)$ A1

Intersects curve again where $\dfrac{2x-5}{x-1} = -\dfrac{1}{3}(x-2) - 1$ M1

$6x - 15 = (x-1)(-x-1)$
$x^2 + 6x - 16 = 0$
$(x-2)(x+8) = 0$
So $x = -8$ A1

[6 marks]

d Sets $y = \dfrac{2x-5}{x-1}$ and rearranges to make x the subject   M1

$xy - y = 2x - 5$

$x(y-2) = y - 5$

$x = \dfrac{y-5}{y-2}$ A1

$f^{-1}(x) = \dfrac{x-5}{x-2}$ A1

[3 marks]

e **i** $x \neq 2$ A1
    ii $f^{-1}(x) \neq 1$ A1

[2 marks]
Total [16 marks]

10 a
$$e^x = 6 - 5e^{-x}$$

$$e^{2x} = 6e^x - 5$$

$$e^{2x} - 6e^x + 5 = 0 \qquad \text{A1}$$

$$\left(e^x - 1\right)\left(e^x - 5\right) = 0 \qquad \text{(M1)}$$
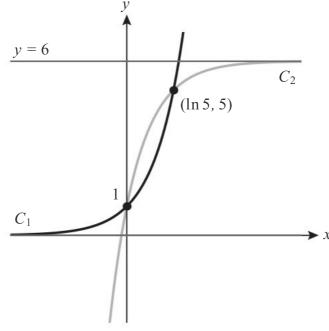
Note: M1 for attempt to solve as quadratic.
$$e^x = 1 \text{ or } e^x = 5$$
$$x = 0 \text{ or } x = \ln 5 \qquad \text{A1A1}$$

[4 marks]

b

$y = 6$

C_2

$(\ln 5, 5)$

1

C_1

$y = e^x$ correct	A1
$y = 6 - 5e^{-x}$: asymptote at $y = 6$	A1
Correct shape	A1
Axis intercepts at $(0, 1)$ and $\left(\ln \frac{5}{6}, 0\right)$	A1

[4 marks]

c Area $= \displaystyle\int_0^{\ln 5} 6 - 5e^{-x} - e^x \, dx$

$$= \left[6x + 5e^{-x} - e^x \right]_0^{\ln 5} \qquad \text{M1A1}$$

Note: M1 for attempt to integrate AND correct limits

$$= \left(6\ln 5 + 5e^{-\ln 5} - e^{\ln 5} \right) - \left(0 + 5 - 1 \right) \qquad \text{(M1)}$$

$$= 6\ln 5 + 5e^{\ln \frac{1}{5}} - e^{\ln 5} - 4$$

$$= 6\ln 5 + 5 \times \frac{1}{5} - 5 - 4 \qquad \text{(M1)}$$

Note: M1 for attempt to simplify e terms
$$= -8 + 6\ln 5$$
that is, $a = -8$, $b = 6$ \qquad A1

[5 marks]
Total [13 marks]

Practice Set B: Paper 2 Mark scheme

SECTION A

1 $\frac{1}{2}r^2\theta = 15$ (A1)

$2r + r\theta = 4r\theta$

$\theta = \frac{2}{3}$ A1

Substituting their θ into their area equation

$\frac{1}{2}r^2\left(\frac{2}{3}\right) = 15$ (M1)

$r = \sqrt{45} = 6.71$ cm   A1

Total [4 marks]

2 a $m = \frac{\ln x^2 - 0}{x - 1}$ (M1)

[2 marks]

$= \frac{\ln x^2}{x - 1}$ A1

b   Finds sequence of values of m for values of x that approach 1.   M1
m tends towards 2   A1

[2 marks]

c   The gradient of the function $\mathrm{f}(x) = \ln x^2$ ...   A1
... at $x = 2$ A1

[2 marks]
Total [6 marks]

3   Let $n = 2k$, where $k > 1$ is an integer   M1
$2^n - 1 = 2^{2k} - 1$

$= \left(2^k - 1\right)\left(2^k + 1\right)$ A1

Since $k > 1$, $2^k - 1 > 1$ R1

Also, $2^k + 1 > 1$ R1
So $2^n - 1$ has two factors both larger than 1 and so is not prime.   R1

Total [5 marks]

4 a $30 = \frac{2\pi}{b}$ so $b = \frac{\pi}{15}$ A1

When $t = 0$, $h = 2$ so $2 = a\cos 0 + c$ (M1)

$2 = a + c$
When $t = 15$, $h = 122$ so $122 = a\cos\left(\frac{\pi}{15} \times 15\right) + c$ (M1)

$122 = c - a$
Solving simultaneously, $a = -60$, $c = 62$ A1A1

[5 marks]

b $50 = -60\cos\left(\frac{\pi}{15}t\right) + 62$

(M1)

From GDC, $t = 6.54, 23.5$ (A1)
So time above $50\,\mathrm{m}$ is 16.9 minutes.   (A1)

[3 marks]
Total [8 marks]

5   **a** $a\left(\dfrac{1-r^3}{1-r}\right)=74\ldots(1)$ A1

         $\dfrac{a}{1-r}=128\ldots(2)$ A1

         Dividing (1) by (2) or substituting   (M1)

         $1-r^3=\dfrac{74}{128}$

         $r^3=\dfrac{27}{64}$

         $r=\dfrac{3}{4}$ A1

         [4 marks]

   b $a=32$ A1

         $S_6=32\left(\dfrac{1-\left(\dfrac{3}{4}\right)^6}{1-\dfrac{3}{4}}\right)$ (M1)

         $=105$ A1

         [3 marks]

         Total [7 marks]

6 $\log_2 x+3=\dfrac{\log_2 a}{\log_2 x}$ (M1)

       $\left(\log_2 x\right)^2+3\log_2 x=\log_2 a$ A1

       EITHER

       From GDC, $y=\left(\log_2 x\right)^2+3\log_2 x$ has a minimum point

       at $(0.354,\,-2.25)$ M1A1

       So for two solutions, $\log_2 a>-2.25$ R1

       $a>2^{-2.25}=0.210$ A1

       OR

       $\left(\log_2 x\right)^2+3\log_2 x-\log_2 a=0$

       Use of discriminant: $3^2-4\left(-\log_2 a\right)>0$ M1A1

       $\log_2 a>-2.25$ (M1)

       $a>2^{-2.25}=0.210$ A1

       Total [6 marks]

7   **a**   **i**   Systematic sampling   A1

         ii   Not all samples are possible, for example, adjacent people on the list
             cannot be chosen.   A1

         [2 marks]

   b   **i**   Men $=\dfrac{46}{46+63}\times 12=5.06$ M1

         So, 5 men   A1

         ii   Simple random sampling   A1

         iii   Uses convenience sampling rather than simple random sampling
             to select the participants in each group.   A1

         [4 marks]

   c $r=0.787$ A1

         Reasonable positive correlation between height and weight; as one increases,
         so does the other.   A1

         [2 marks]

   d   **i** $h=0.769w+118$ A1

         $h=0.769\times 68+118=170\,\text{cm}$ A1

         ii $w=0.806h-70.0$ A1

         $w=0.806\times 170-70.0=67.0\,\text{kg}$ A1

         [4 marks]

   e   For example, take a larger sample; create separate regression lines for men
         and women   A1A1

         [2 marks]

         Total [14 marks]

SECTION B

8   **a** $X \sim \mathrm{B}(10, 0.04)$ (M1)

       $\mathrm{P}(X = 2) = 0.0519$ A1

       [2 marks]

   b $\mathrm{P}(X \geqslant 2) = 1 - \mathrm{P}(X \leqslant 1)$ (M1)

        $= 0.0582$ A1

       [2 marks]

   c **i** $0.04n = 2$ (M1)

          $n = 50$ A1

      ii $\mathrm{Var}(X) = 50 \times 0.04 \times 0.96$ (M1)

           $= 1.92$ A1

       [4 marks]

   d $X \sim \mathrm{B}(5, 0.0582)$ (M1A1)

       Note: M1 for use of binomial with $n = 5$

       $\mathrm{P}(X > 1) = 1 - \mathrm{P}(X \leqslant 1)$ (M1)

       $= 0.0301$ A1

       [4 marks]

   e $\mathrm{P}(X \geqslant 1) > 0.95$

       $1 - \mathrm{P}(X = 0) > 0.95$ (M1)

       $\mathrm{P}(X = 0) < 0.05$ (M1)

       $0.96^n < 0.05$ A1

       $n > 73$ A1

       That is, smallest number of calls is $n = 74$

       [4 marks]

       Total [16 marks]

9 **a** $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \cos x\mathrm{e}^{\cos x} + \sin x\left(-\sin x\mathrm{e}^{\cos x}\right)$ (M1)A1

Note: M1 for attempt at product rule

$\mathrm{e}^{\cos x}\left(\cos x - \sin^2 x\right) = 0$

$\sin^2 x = \cos x$ M1

$\dfrac{\sin^2 x}{\cos x} = 1$

$\sin x\dfrac{\sin x}{\cos x} = 1$ A1

$\sin x\tan x = 1$ AG

[4 marks]

b $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\sin x\mathrm{e}^{\cos x}\left(\cos x - \sin^2 x\right) + \mathrm{e}^{\cos x}\left(-\sin x - 2\sin x\cos x\right)$ (M1)A1

Note: M1 for attempt at product rule on their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

$\sin x\mathrm{e}^{\cos x}\left(\sin^2 x - \cos x - 1 - 2\cos x\right) = 0$

Use of $\sin^2 x = 1 - \cos^2 x$

$\sin x\left(1 - \cos^2 x - 3\cos x - 1\right) = 0$

$-\sin x\cos x\left(\cos x + 3\right) = 0$ M1

Since $\cos x + 3 \neq 0$, $\sin x\cos x = 0$ R1

$\dfrac{1}{2}\sin 2x = 0$ A1

$\sin 2x = 0$ AG

[5 marks]

c For *x*-axis intercepts: $\sin x\mathrm{e}^{\cos x} = 0$

$\sin x = 0$

$x = 0, \pi$ A1

So, area $= \displaystyle\int_0^\pi \sin x\,\mathrm{e}^{\cos x}\,\mathrm{d}x$ (M1)A1

$= [-\mathrm{e}^{\cos x}]_0^\pi$

Note: M1 for $k\mathrm{e}^{\cos x}$

$= (-\mathrm{e}^{\cos \pi}) - (-\mathrm{e}^{\cos 0})$ M1

$= -\mathrm{e}^{-1} + \mathrm{e}$

that is, $a = -1$, $b = 1$ A1

[5 marks]

Total [14 marks]

Practice Set C: Paper 1 Mark scheme

SECTION A

1 **a** Use of $640 = \dfrac{20}{2}\left(7 + u_{20}\right)$ or $640 = \dfrac{20}{2}\left(14 + 19d\right)$ (M1)

  57   A1

  [2 marks]

b $19d = 50$ or $u_{39} - u_{20} = u_{20} - u_1$ (M1)

  107   A1

  [2 marks]

  Total [4 marks]

2 **a** $\dfrac{1}{2} \times 10^2 \times \theta = 75$ M1

  $\theta = 1.5$ A1

  [2 marks]

b $10 \times$ their θ M1

  $+\ 20$ (M1)

  $=\ 35\,\mathrm{cm}$ A1

  [3 marks]

  Total [5 marks]

3 **a** $(90, 88)$ (M1)

  $200 - 88 = 112$ A1

  [2 marks]

b 15% of $200 = 30$ (M1)

  Line at 170 on graph crosses at $(104, 170)$ B1

  $104\,\mathrm{g}$ A1

  [3 marks]

  Total [5 marks]

4 **a** Translation 3 units to the right   A1

  Vertical stretch with scale factor 2   A1

  [2 marks]

b $k \times 3^{2x}$ (M1)

  12×3^{2x} (M1)

  $12 \times 9^x\,(A = 12, B = 9)$ A1A1

  [4 marks]

  Total [6 marks]

5 $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 2ax$ (M1)

  $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 12x - 2a$ A1

  $=\ 0$ M1

  $a = 6$ A1

  $-6 = 16 - 4 \times$ their $a + 2 + 2b$ M1A1

  $b = 0$ A1

  Total [7 marks]

6 $y = \mathrm{e}^x$ (does not need to be explicitly used)   (M1)

  $\left(y - 4\right)\left(y + 2\right) = 0$ M1

  $\mathrm{e}^x = 4$ or -2 A1A1

  $\mathrm{e}^x = -2$ is impossible   B1

  $x = \ln 4$ A1

  Total [6 marks]

7   Obtain $k \sin (x^2)$ M1

     $4 \sin (x^2)$ A1

     Attempt to use $x = 0$ and $x = \sqrt{\dfrac{\pi}{3}}$ and subtract   (M1)

     $\sin \left(\dfrac{\pi}{3}\right) - \sin (0)$ A1

     Using $\sin \left(\dfrac{\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$ B1

     $2\sqrt{3}$ A1

     Total [6 marks]

SECTION B

8   **a** $0.3 \times 9 - 2.4 \times 3 + 3.6 = -0.9$ M1

         $0.9\,\mathrm{m\,s^{-1}}$ M1

         [2 marks]

   b   Attempt to find $\dfrac{\mathrm{d}v}{\mathrm{d}t}$ M1

         $0.6t - 2.4 = 2.4$ A1

         $8\,\mathrm{s}$ A1

         [3 marks]

   c $2\,\mathrm{s}$ and $6\,\mathrm{s}$ B1

         [1 mark]

   d $\int v\,\mathrm{d}t = 0.1t^3 - 1.2t^2 + 3.6t$ M1A1

       i   use limits 0 and 2   M1

           $3.2\,\mathrm{m}$ A1

       ii   use limits 2 and 3 to get -0.5 M1

           $3.2 + 0.5 = 3.7\,\mathrm{m}$ A1

         [6 marks]

   e   their 3.2 – their 0.5 or $\int_0^3 v\,\mathrm{d}t$ M1

         $2.7\,\mathrm{m}$ A1

         [2 marks]

         Total [14 marks]

9   **a** $[4 - x^2 = 4 - x] \Rightarrow x^2 = x$ M1

         $x = 0,\ 1$ A1

         $A(0, 4).\ B(1, 3)$ A1

         [3 marks]

   b $(0.5, 3.5)$ A1

         [1 mark]

   c   Attempt to solve $4 - x^2 = k - x$ (M1)

         Obtain quadratic $x^2 - x + (k - 4) = 0$ A1

         Consider the discriminant: $1 - 4(k - 4)$ M1

         Set it $= 0$ M1

         $k = \dfrac{17}{4}$ A1

         [5 marks]

   d   Attempt to solve $x^2 - x + \dfrac{1}{4} = 0$ or use $x = -\dfrac{b}{2a}$ (M1)

         $x = \dfrac{1}{2}$ A1

         [2 marks]

   e $D\left(\dfrac{1}{2}, \dfrac{15}{4}\right)$ B1

         $\dfrac{1}{4}$ A1

         [2 marks]

         Total [13 marks]

10 a i   Use quotient rule (correct structure, condone wrong +/− signs)   M1

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(\cos x + \sin x)(\sin x + \cos x) - (\sin x - \cos x)(\cos x - \sin x)}{(\sin x + \cos x)^2}$$ A1

$$= \frac{2\sin^2 x + 2\cos^2 x}{\sin^2 x + \cos^2 x + 2\sin x \cos x}$$ A1

At least one use of $\sin^2 x + \cos^2 x = 1$ B1
Use of $2\sin x \cos x = \sin 2x$ B1
Completely correct derivation   A1

ii   0 to 2   B1
$1 + 2\sin 2x \neq 0$ for $x \in [0, \frac{\pi}{2}]$ so $f'(x) > 0$ for all $x \in [0, \frac{\pi}{2}]$ A1

[8 marks]

b   Differentiate again and set = 0   M1

$$f''(x) = \frac{-4\cos 2x}{\left(1 + \sin 2x\right)^2}$$ A1

$$2x = \frac{\pi}{2}$$ (M1)

$$\left(\frac{\pi}{4}, 0\right)$$ A1

[4 marks]

c   graph A   A1
For example, increasing   A1

[2 marks]
[Total 14 marks]

Practice Set C: Paper 2 Mark scheme

SECTION A

1 **a**   midpoints: 10, 13.5, 17.5, 22.5, 26.5   (M1)
     mean = 17.8   A1
     SD = 5.40   A1
[3 marks]

   b   '17.8' × 2.54   (M1)
     mean = 45.3   A1
     variance = 188   A1
[3 marks]
Total [6 marks]

2 **a** $\dfrac{18^2\pi}{2} \times 83$ (M1)
     $4.22 \times 10^4\,\text{cm}^3$ A1A1
[3 marks]

   b $\dfrac{4}{3}\pi r^3$ = their volume   M1
     $r^3 = \dfrac{\text{volume} \times 3}{4\pi}$ (M1)
     $r = 21.6\,\text{cm}$ A1
[3 marks]
Total [6 marks]

3 **a** $9.1^2 = 6.8^2 + 4.7^2 - 2(6.8)(4.7)\cos B$ (M1)
     $\cos B = -0.227$ (A1)
     $B = 103°$ A1
[3 marks]

   b $\dfrac{1}{2}(6.8)(4.7)\sin(\text{their } B)$ M1
     $15.6\,\text{cm}^2$ A1
[2 marks]
Total [5 marks]

4 **a**   At least one *x*-coordinate correct   M1
     Both *x*-coordinates correct   A1
     $(-0.175, 1.01)$ and $(1.85, 1.93)$ given as coordinates   A1
[3 marks]

   b $\displaystyle\int\!\!\!\int\left(4\mathrm{e}^{-(x-1)^2} - \dfrac{x}{\sin x}\right)\mathrm{d}x$ M1
     Use *x*-coordinates from part **a** as limits   M1
     3.44   A1
[3 marks]
Total [6 marks]

5 $d\left[= \dfrac{1+3}{2}\right] = 2$ B1
     $\text{period} = \dfrac{2\pi}{3}$ (M1)
     $b = 3$ A1
     e.g. $x = 0, y = 1$: $2 + \sin(3(-c)) = 1$ M1
     $c = \dfrac{\pi}{6}$ A1
Total [5 marks]

6 a increases: 8, 10, 7, 8, 7   (M1)
average = 8   A1
[2 marks]

b 26 + 11 × their average   M1
114   A1
[2 marks]

c $y = 8.06x + 18.5$ A1
115   A1
[2 marks]
Total [6 marks]

7 a $P(A|B) \neq P(A)$ A1
[1 mark]

b $P(A \cap B) = P(A|B)P(B) \left[= \frac{2}{3} P(A)\right]$ M1

use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ M1

$\frac{1}{5} = P(A) + \frac{1}{6} - \frac{2}{3} P(A)$ A1

$P(A) = \frac{1}{10}$ (A1)

$P(A \cap B) = \frac{1}{15}$ A1

[5 marks]
Total [6 marks]

Alternative for **b**:
Draw a Venn diagram with $\frac{1}{30}, x, \frac{1}{6} - x$ M1A1

$\frac{x}{1/6} = 4 \left(x + \frac{1}{30}\right)$ B1

$x = \frac{1}{15}$ M1A1

SECTION B

8 a $\Phi^{-1}(0.88) = 1.175$ B1

$\frac{8.5 - \mu}{2.5} =$ positive *z*-value   M1

$\mu = 5.56$ A1
[3 marks]

b 0.927   A1
[1 mark]

c $\frac{P(X < 8.3)}{\text{answer } \mathbf{b}}$ M1

0.931   A1
[2 marks]

d 20 × answer **b**   (M1)
18.5   A1
[2 marks]

e Using B(20, their 0.927)   M1
$1 - P(X \leq 17)$ M1
0.824   A1
[3 marks]

f using answer from **e**   (M1)
$2 \times 0.824 \times (1 - 0.824)$ M1
0.290   A1
[3 marks]
Total [14 marks]

9 **a** $5^2 + 5^2 + 5^2 \; [= 75]$ M1
8.66 cm   A1

[2 marks]

b $\sin^{-1}\left(\dfrac{5}{8.66}\right)$ or $\tan^{-1}\left(\dfrac{5}{\sqrt{50}}\right)$ M1

65.9° or 1.15 rad   A1

[2 marks]

c $\pi r^2 h = 125$ M1

$h = \dfrac{125}{\pi r^2}$ A1

SA $= 2\pi rh + 2\pi r^2$, replace h by $\dfrac{125}{\pi r^2}$ M1

Simplify $2\pi r \times \dfrac{125}{\pi r^2}$ to $\dfrac{250}{r}$ A1

[4 marks]

d Graph of $y = \dfrac{250}{x} + 2\pi x^2$ M1
Minimum value is 138   A1
The surface area of the cylinder is smaller (138 vs. 150)   A1

[3 marks]

e $r = 2.71$, $h = \dfrac{125}{\pi r^2} = 5.42$ B1

$\tan \theta = \dfrac{5.42}{2 \times 2.71}$ M1

$\theta = 45°$ [0.785 radians]   A1

[3 marks]
Total [14 marks]

10 **a** $1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}$ B1

[1 mark]

b $\dfrac{1}{1 - \frac{1}{2}}$ M1

$= 2$ A1

[2 marks]

c 1, 1.33,   A1
1.44, 1.48   A1

[2 marks]

d Find further terms   (M1)
1.5   A1

[2 marks]

e Geometric series, $u_1 = \dfrac{2}{3}$, $r = \dfrac{1}{3}$ (M1)

$w_n = \dfrac{\dfrac{2}{3}\left(1 - \left(\dfrac{1}{3}\right)^n\right)}{1 - \dfrac{1}{3}}$ M1

$= 1 - \left(\dfrac{1}{3}\right)^n$ A1

[3 marks]

f Use S_∞ formula or state $\left(\dfrac{1}{3}\right)^n \to 0$ or find further terms   (M1)

1   A1

[2 marks]
Total [12 marks]

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